JITTER EFFECTS IN CONTINUOUS-TIME ΣΔ MODULATORS WITH DELAYED RETURN-TO-ZERO FEEDBACK

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ABSTRACT

A general method to design continuous-time Sigma–Delta (ΣΔ) modulators with delayed return-to-zero (RZ) feedback pulse is presented. This method can be applied to design modulators with any arbitrary delay and pulse width. It is shown that the key-point to avoid the degrading effects of clock jitters, is to generate a low jitter feedback pulse-width, using a monostable multivibrator.

1. INTRODUCTION

Recently, continuous-time ΣΔ modulators have received increasing attention for either high-speed [1] or low-power [2] applications. The main advantages of continuous-time ΣΔ modulators over their discrete-time counterparts are: higher sampling rate [3], lower thermal noise [4] and intrinsic anti-aliasing filtering [5, 6].

The first major problem encountered in the implementation of continuous-time ΣΔ modulators is the quantizer delay. In a classical continuous-time ΣΔ modulator [7], the D/A is driven directly by the quantizer output. The D/A output, which constitutes the feedback signal, is held constant during a complete clock period (NRZ scheme). This structure suffers from excess loop delay which is due to non-zero response time of the quantizer. Although the problem related to the constant component of the quantizer delay may be solved by a multi feedback structure[8], the signal dependent component of the quantizer delay would continue to degrade the ΣΔ performance[9]. The second major problem encountered in the implementation of continuous-time ΣΔ modulators is clock jitter. Timing errors due to clock jitter in the feedback loop increases the noise level in the signal band [2, 10]. In the continuous time ΣΔ modulator with delayed RZ feedback pulse, clock jitter modifies both the pulse-delay and the pulse-width.

Here, we propose a structure including an explicit delay in the feedback loop with a return to zero pulse (RZ scheme) [5]. This additional delay t_d allows for a complete settling of the quantizer output. In this structure, the coefficients are chosen so as to take into account the delay and the D/A pulse-width τ in the feedback loop. Then, the impact of the two types of jitter on the system performance is studied and it is shown that the jitter modulating the pulse-width has a more degrading effect on SNR.

2. DESIGN PROCEDURE

A general structure of a single-bit Nth order continuous time ΣΔ low-pass modulator is shown in Fig. 1. The D/A ouput signal is supposed to have a rectangular shape with a pulse-width of τ delayed by an amount of t_d with respect to the sampling instant, Fig. 2. Because of using a single-bit quantizer and an RZ-scheme, the feedback signal takes three values +1,-1 and 0.

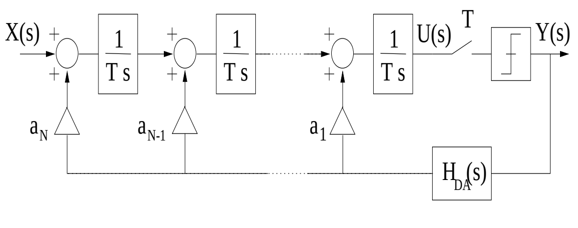

Figure 1: *A general structure of an Nth order single-bit continuous-time ΣΔ modulator.*

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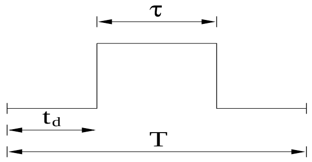

Figure 2: *The feedback D/A pulse $H_{DA}(s)$.*

The coefficients a_n are calculated so that the continuous-time system presents the same quantization noise-transfer function as its discrete-time equivalent [6, 5]. The loop-gain of the modulator is given by:

$$\begin{aligned} L.G &= H_{DA}(s)K(s) \\ &= H_{DA}(s)\left(\frac{a_1}{Ts} + \frac{a_2}{T^2s^2} + \ldots + \frac{a_N}{T^N s^N}\right) \end{aligned} \quad (1)$$

where $K(s)$ is the loop filter and $H_{DA}(s)$ is the transfer function of the D/A. In order to be able to use modified z-transform [11], we define the parameters m_1 and m_2 as : $t_d = (1-m_1)T$ and $t_d + \tau = (1-m_2)T$, where T represents the sampling period. So, the transfer function of the D/A is obtained :

$$H_{DA}(s) = \frac{e^{-Ts}}{s}(e^{m_1 Ts} - e^{m_2 Ts}) \quad (2)$$

It can be easily verified that the z-transform of the loop-gain is calculated from:

$$\mathcal{Z}(L.G) = \mathcal{Z}_{m_1}\left(\frac{K(s)}{s}\right) - \mathcal{Z}_{m_2}\left(\frac{K(s)}{s}\right) \quad (3)$$

In the following, the above relation will be used to find the coefficients for the first and second order systems.

2.1. First Order System

For a first-order system, equation (3) gives :

$$\begin{aligned} \mathcal{Z}(L.G) &= \frac{a1}{T}\left(\mathcal{Z}_{m_1}\left(\frac{1}{s^2}\right) - \mathcal{Z}_{m_2}\left(\frac{1}{s^2}\right)\right) \\ \mathcal{Z}(L.G) &= a_1 \frac{\tau}{T}\frac{z^{-1}}{1-z^{-1}} \end{aligned} \quad (4)$$

Equating the above relation to the loop gain of a discrete-time system, given by $-z^{-1}/(1-z^{-1})$, the coefficient is obtained as $a_1 = -T/\tau$. This shows that a_1 is independent of the pulse delay time t_d, and that any variations in t_d do not modify the noise transfer function as long as the pulse-width τ remains constant.

2.2. Second Order System

In the case of the second-order system, the z-transform of the loop gain is obtained using (3) :

$$\begin{aligned} \mathcal{Z}(L.G) &= \frac{a2}{T^2}\left(\mathcal{Z}_{m_1}\left(\frac{1}{s^3}\right) - \mathcal{Z}_{m_2}\left(\frac{1}{s^3}\right)\right) + \frac{a1}{T}\left(\mathcal{Z}_{m_1}\left(\frac{1}{s^2}\right) - \mathcal{Z}_{m_2}\left(\frac{1}{s^2}\right)\right) \\ \mathcal{Z}(L.G) &= a_2(m_1 - m_2)\frac{z^{-2}}{(1-z^{-1})^2} \\ &\quad + \left(\frac{a_2}{2}(m_1^2 - m_2^2) + a_1(m_1 - m_2)\right)\frac{z^{-1}}{1-z^{-1}} \end{aligned} \quad (5)$$

Equating the above relation to the loop gain of a second-order discrete-time system, given by $(-2z^{-1} + z^{-2})/(1-z^{-1})^2$, the coefficients a_1 and a_2 are obtained, respectively, as:

$$\begin{aligned} a_1 &= (m-2)T/\tau \\ a_2 &= -T/\tau \end{aligned} \quad (6)$$

where we have defined: $m = (m_1 + m_2)/2$. It is seen that the coefficient a_2 is independent of the delay time t_d but a_1 depends on t_d through the parameter m. It is clear that for the NRZ case, i.e., $\tau = T$ and $t_d = 0$, the above relations will give the same results obtained by Candy [7]: $a_1 = 1.5$ and $a_2 = 1$. Relation (6) gives the modulator coefficients for any desired value of τ and t_d. However, practical considerations may limit the degrees of freedom. A pulse-delay of $t_d = 0.5T$ is directly realizable using the clock pulse. An arbitrary pulse-width can be generated using a monostable multivibrator circuit. Such circuit offers the advantage of lowering the pulse-width clock jitter. This is an important point because, as will be discussed in the following section, the pulse width jitter has a more degrading effect than the pulse-delay jitter.

3. CLOCK JITTER EFFECT

In this section we consider the random variation of the D/A output pulse due to clock jitters. The delay clock jitter $\overline{\delta t_d^2}$ and the pulse-width clock jitter $\overline{\delta \tau^2}$ give rise to random variations at the integrators output. This introduces an additional noise component at the modulator output which decreases the SNR. The simulations are performed for the following nominal values: $t_d = 0.5T$ and $\tau = 0.4T$. For the second-order system, we use (6) to get the modulator coefficients : $a_1 = -4.25$ and $a_2 = -2.5$. Continuous-time simulations have been carried-out using the electrical simulator ELDO. The two clock jitters are assumed to be white and uncorrelated with a Gaussian distribution.

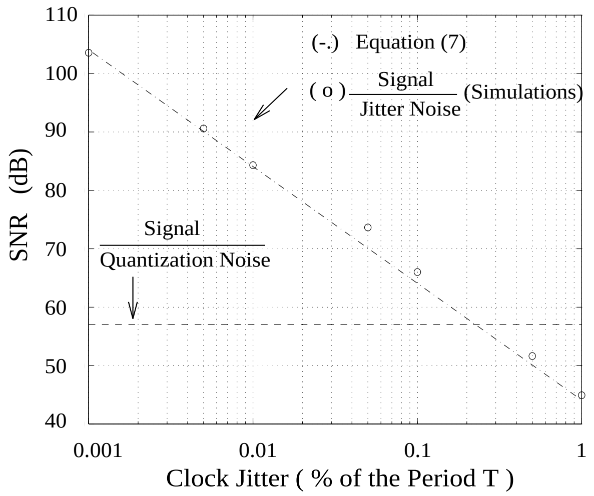

Figure 3: *SNR_Q and SNR_J against relative pulse-width clock jitter $\sqrt{\overline{\delta\tau^2}}/T$.*

3.1. First Order System

In the first-order modulator, the sampled output does not depend on t_d, so the clock jitter $\overline{\delta t_d^2}$ is expected to cause no SNR loss. However, the clock jitter $\overline{\delta\tau^2}$ gives rise to white noise at the output. A clock variation $\sqrt{\overline{\delta\tau^2}}$ causes an amplitude error of $\sqrt{\overline{\delta\tau^2}}/\tau$ giving a noise power of $\overline{\delta\tau^2}/\tau^2$. Since the ouput noise due to clock jitter $\overline{\delta\tau^2}$ is white, the output Signal to Jitter-Noise Ratio (SNR_J) for an input sine-wave of amplitude α is found to be:

$$SNR_J = 10\log\frac{\alpha^2 OSR}{2\overline{\delta\tau^2}/\tau^2} \qquad (7)$$

In Fig. 3, the above relation is plotted along with the Signal to Quantization-Noise Ratio SNR_Q for the previously mentioned nominal values and $OSR = 128$. In an optimal design, the clock jitter noise must be kept below the quantization noise. To obtain the output Signal to Jitter-Noise Ratio (SNR_J) by simulation, the quantizer was removed from the $\Sigma\Delta$ loop. The result of simulations for the total Signal to Noise Ratio ($SNR = SNR_Q + SNR_J$) are shown in Fig. 4. It is seen that as predicted, the clock jitter $\overline{\delta t_d^2}$ has no effect on SNR but the effect of $\overline{\delta\tau^2}$ becomes significant for $\sqrt{\overline{\delta\tau^2}}$ greater than 0.1% of the clock period.

3.2. Second Order System

In order to study the output spectrum of the second-order modulator affected by clock jitter, the quantizer was removed from the $\Sigma\Delta$ loop. In this case, the output contains only the input signal and the clock jitter resulted noise. Fig. 5 shows the ouput spectrum due to clock jitters $\overline{\delta\tau^2}$ and $\overline{\delta t_d^2}$ respectively. It is seen that the jitter $\overline{\delta\tau^2}$ produces an almost white noise at the output while $\overline{\delta t_d^2}$ gives rise to a first-order shaped noise. In other words, the resulted noise is high-pass filtered by the function $(1 - z^{-1})$. In fact, the noise sources due to clock jitters $\overline{\delta\tau^2}$ and $\overline{\delta t_d^2}$ can be modeled as shown in Fig. 6. The clock jitter $\overline{\delta\tau^2}$ appears directly at the output while the clock jitter $\overline{\delta t_d^2}$ is submitted to a first-order noise-shaping. This means that in a second-order modulator, SNR loss is mainly related to the random variations of the pulse width $\overline{\delta\tau^2}$.

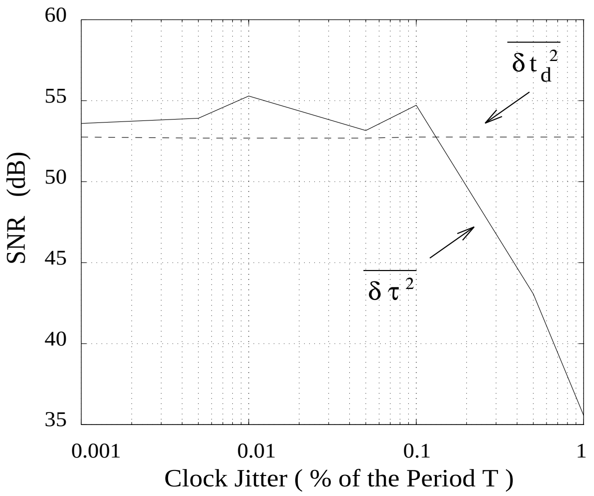

Figure 4: *The effect of clock jitters, $\overline{\delta\tau^2}$ and $\overline{\delta t_d^2}$, on the SNR of a first order system, (input signal = $-6dB$, $OSR = 128$).*

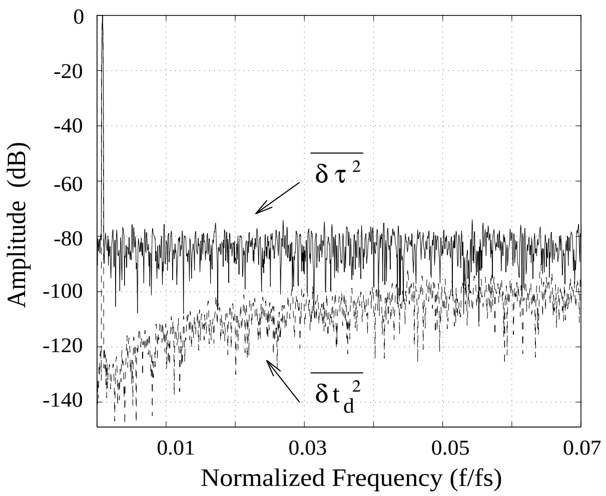

Figure 5: *The effect of clock jitters, $\overline{\delta\tau^2}$ and $\overline{\delta t_d^2}$, on the output spectrum of a second order system, (quantizer removed).*

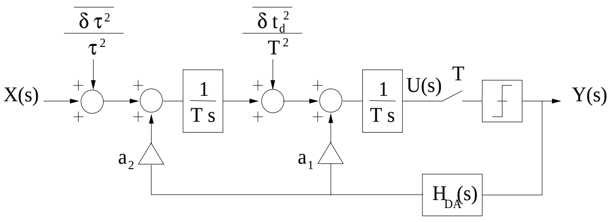

Figure 6: *A second order continuous-time $\Sigma\Delta$ model taking into account the clock jitters $\overline{\delta\tau^2}$ and $\overline{\delta t_d^2}$.*

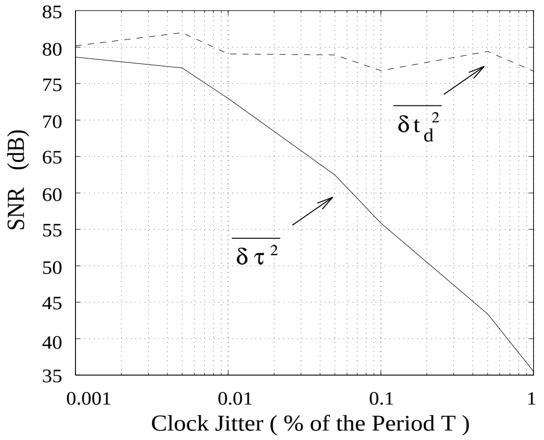

Figure 7: *Effect of clock jitters, $\overline{\delta\tau^2}$ and $\overline{\delta t_d^2}$, on the SNR of a second order system, (input signal $= -6dB$, $OSR = 128$).*

Fig. 7 compares the impact of clock jitters $\overline{\delta\tau^2}$ and $\overline{\delta t_d^2}$ on the signal-to-noise ratio of a second-order modulator operating with an input amplitude of $-6dB$ below the feedback level and an $OSR = 128$. It is seen that SNR is subject to a severe deterioration even for a $\sqrt{\overline{\delta\tau^2}}$ as low as 0.01% of the clock period while increasing $\sqrt{\overline{\delta t_d^2}}$ from 0.01% to 1% causes only a loss of $-2.3dB$ in SNR.

In this section, it has been shown that a first-order modulator is insensitive to clock jitter modifying the pulse-delay, while the random variations of pulse-width give rise to an output white noise. In a second-order modulator, the output noise due to variations of feedback pulse-delay has a first-order high-pass filtered form. On the other hand, the clock jitter modifying the feedback pulse-width generates an almost white noise.

4. CONCLUSION

In this paper, it was proposed to insert an explicit delay in the feedback loop of continuous-time $\Sigma\Delta$ modulators in order to give sufficient time for the quantizer to settle. This extra delay may be taken into account in the calculation of the coefficients of the modulator so that the overall noise transfer function is not modified. A method based on modified z-transform was presented to obtain the modulator coefficients for any feedback pulse delay and duration. A study of clock jitters has shown that the output noise due to random variations of feedback pulse-width has a more significant degrading effect on SNR. This makes clear that the key-point to realize high-performance continuous-time sigma-delta modulators is to generate low-jitter feedback pulse-width. This may be achieved through an integrated monostable multivibrator with stablized pulse-width.

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