Analog Fault Detection based on Statistical Analysis

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ABSTRACT: In analog circuits, process variations result in physical parameter variations. Simulated values must then be considered with there tolerance intervals. Consequently, contrarily to digital circuits where the outputs are either '0' or '1' such that we can decide without ambiguity whether a fault is detectable or not, for analog circuits the fault detectability is a vague problem as the fault can either be completely detectable, partially detectable or completely undetectable which makes it very difficult to take a decision. In order to solve this decision problem, we have introduced the probability to detect fault (PDF) function which allows to formalize the problem of analog fault detection under parameter variations.

1. Introduction

The growing need of analog and mixed analog-digital circuits has motivated the research in design and test tools of complex analog circuits. Even if the area of the analog part in a mixed circuit represents less than 15% of the global area, the test of this analog part contributes significantly to the global manufacturing cost of the mixed circuit. Traditionally, analog circuits are tested by functional testing which is known to be costly and not efficient. In order to improve the quality and reduce the cost of analog circuit testing, recent research addresses the area of structural testing of analog circuits [1] and [2].

Usually, we use a fault-based technique to test analog circuits [3] and [4]. But due to process parameter deviations, measured performances are to be considered with their tolerance intervals. Deviations from the nominal value are caused by many factors such as manufacturing processes, model approximations, aging and surrounding environment. Using only the nominal values to compute analog fault coverage and evaluate the effectiveness of such a set of stimuli can result in a significant difference with the reality. So to have an accurate analog fault detection, we need to consider the effect of tolerances on the circuit specifications. The most widely used approach to take into account parameter variations is the Monte Carlo simulation [5], [6] and [7]. The other approaches are : interval simulations [8] and [9] and constraint logic programming [10]. With those approaches, we compute the intervals of good and faulty circuits and then we compare the two intervals to decide if the fault is detectable or not. Until now, there is no proposed method to quantify by a real number how much a given fault can be detected by a given test with a certain risk α of error probability.

In this work we propose a new approach of fault detection and fault coverage based on the probability to detect a fault (PDF) function. We will also use the PDF function to improve the speed of our fault simulation algorithm [11] by using 2 early stop rules to reduce the number of simulations needed to compute PDF and fault coverage. In this paper we introduce a probability to detect a fault (PDF) function, which is used to get accurate fault detection. This paper is organized as follow : Section II gives some definitions and theorems of the statistical and the probability theory used in this paper. In section III, we introduce the probability to detect a fault *(PDF)* function, and section IV describes the new analog fault simulation based on *PDF* function. The results and conclusions are given in sections V and VI.

2. Statistical and Probability theory

2.1 *Normal distribution*

The normal or Gauss distribution is defined as the distribution with the density

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp \frac{-(x-\mu)^2}{2\sigma^2} \qquad (1)$$

Here, μ is the mean and σ is the standard deviation. For a risk α ($\alpha = 95\%, 99\%, \cdots$) or confidence level $1-\alpha$, we define the corresponding confidence interval $I_x = [\mu - k_\alpha\sigma, \mu + k_\alpha\sigma]$ as the interval satisfying

$$P(\mu - k_\alpha\sigma < X \leq \mu + k_\alpha\sigma) = 1 - \alpha \qquad (2)$$

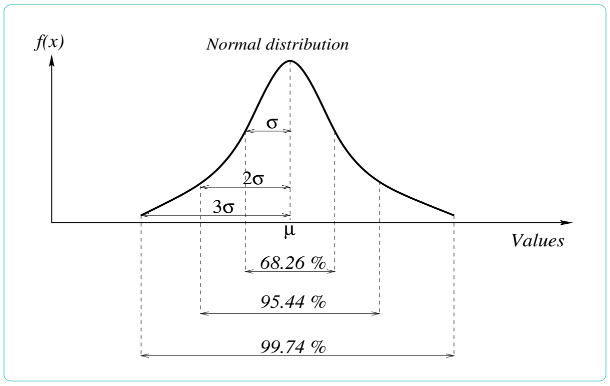

Figure 1: Normal or Gauss distribution.

For a normal distribution, table 1 gives the values of k_α for different risks α (see figure 1). Practically speaking, for a given risk α, this means that all the values will lie between $\mu - k_\alpha\sigma$ and $\mu + k_\alpha\sigma$.

risk α	confidence level $1-\alpha$	k_α
0.10	90%	1.645
0.05	95%	1.960
0.01	99%	2.576

Table 1: k_σ for different risks α.

2.2 *Confidence intervals for μ*

As an approximation of the mean μ and standard deviation σ, we may take the mean $\bar{x}$ and standard deviation S of a corresponding sample:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \quad and \quad s = \sqrt{\frac{(\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} \qquad (3)$$

where n is the size of the sample. Using the knowledge of the t-distribution of *Student*, the confidence interval for μ of the normal distribution with unknown σ is

$$I_\mu = [\bar{x} - t_{\alpha[n-1]}\frac{s}{\sqrt{n}}, \bar{x} + t_{\alpha[n-1]}\frac{s}{\sqrt{n}}] \qquad (4)$$

where $t_{\alpha[n-1]}$ is the *Student* parameter of the t-distribution with risk α and $(n-1)$ degrees of freedom. Table 2 gives some values of $t_{\alpha[n-1]}$ for different values of α and n.

risk	$t_{\alpha[n-1]}$ of *Student*			
α	$(n=5)$	$(n=10)$	$(n=20)$	$(n=30)$
0.10	2.132	1.833	1.729	1.699
0.05	2.776	2.262	2.093	2.045
0.01	4.604	3.250	2.861	2.756

Table 2: $t_{\alpha[n-1]}$ for different values of α and n.

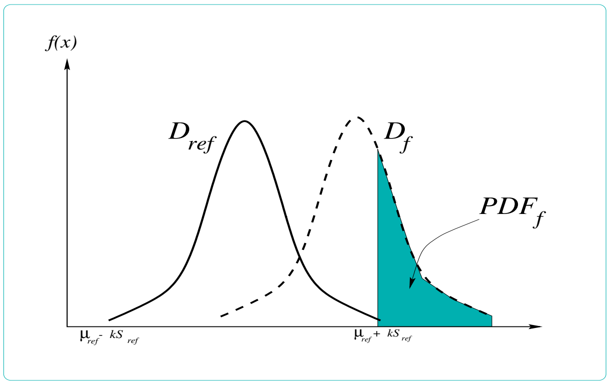

Figure 2: Definition of the probability to detect a fault (PDF).

3. Probability to detect a fault *PDF* function

3.1 *Definition*

The main idea behind the probability to detect a fault (PDF) function is to quantify by a number $\in [0,1]$ how much a fault can be detected by a given test. Let's consider D_{ref} and D_f are the distributions of the output values of the good circuit (reference) and the faulty circuit respectively, and I_{ref} and I_f their correspondent confidence intervals. The PDF function must satisfy the follow three conditions (see figure 3) :

1. If $I_f \subset I_{ref}$ then the fault is completely undetectable and the PDF must be equal to 0.
2. If $I_f \cap I_{ref} = 0$ the fault is completely detectable and the PDF must be equal to 1.
3. Else the fault is partially detectable (or partially undetectable) and $0 <$ PDF < 1.

The PDF function is defined as follows:

$$\boxed{PDF_f = 1 - \int_{I_{ref}} D_f(x)\,dx} \qquad (5)$$

with $I_f = [\mu_{ref} - k_\alpha\sigma_{ref}, \mu_{ref} + k_\alpha\sigma_{ref}]$. Since $\int_{I_{ref}} D_f(x)\,dx$ represents the probability that a faulty value V_f be inside the good distribution D_{ref}, PDF_f represents the probability that a faulty value be outside the good distribution D_{ref}, which is equivalent to the probability that a fault f will be detected (see figure 2). We can easily show that PDF_f verifies the above three conditions. If we assume that a faulty distribution D_f follows a normal distribution with a mean value $\bar{x}_f$ and standard deviation s_f, the PDF function becomes :

$$\boxed{PDF = 1 - \int_{I_{ref}} \frac{1}{\sqrt{2\pi}s_f}\exp\left(-\frac{(x-\bar{x}_f)^2}{2s_f^2}\right)dx} \qquad (6)$$

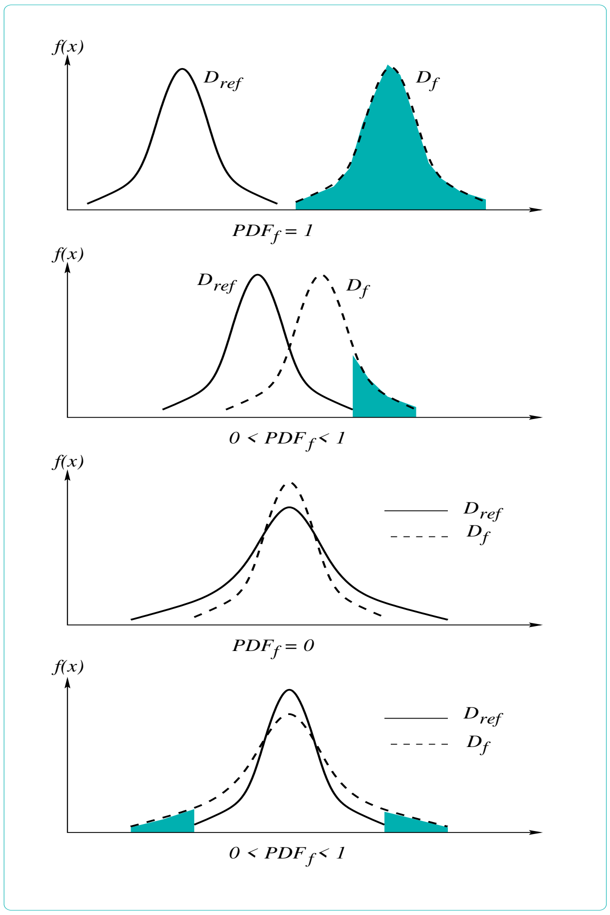

Figure 3: Different PDF values.

3.2 *Computation of PDF*

The problem of formulation 5 is that we need to compute a large number of simulations for each faulty circuit to get the distribution D_f used in 5. In order to reduce the number of simulations, we propose two early stop rules.

3.2.1 First rule

Let V_f represents the current simulated value of the faulty circuit. According to (2) we know that $V_f \in I_f$ ($I_f = [\mu_f - k_\alpha \sigma_f, \mu_f + k_\alpha \sigma_f]$ is the confidence interval of the faulty distribution D_f), and if we assume that $\sigma_f = \sigma_{ref}$, we can show that if

$$\boxed{|\mu_{ref} - V_f| \geq 3k_\alpha \sigma_{ref} \implies PDF_f = 1} \tag{7}$$

So if the simulated value V_f verifies the condition $|\mu_{ref} - V_f| \geq 3k_\alpha \sigma_{ref}$, then we can stop simulations and put $PDF_f = 1$ (see figure 4).

3.2.1 Second rule

If now, we consider the estimated mean $\overline{v}_f$ of the n first simulations. According to (4), we know that

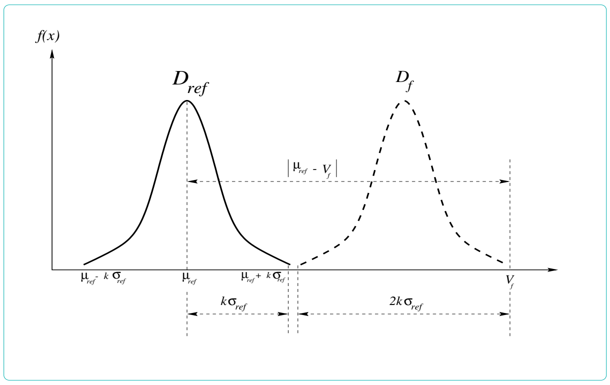

Figure 4: Computation of PDF : case $|\mu_{ref} - V_f| \geq 3k_\alpha \sigma_{ref}$.

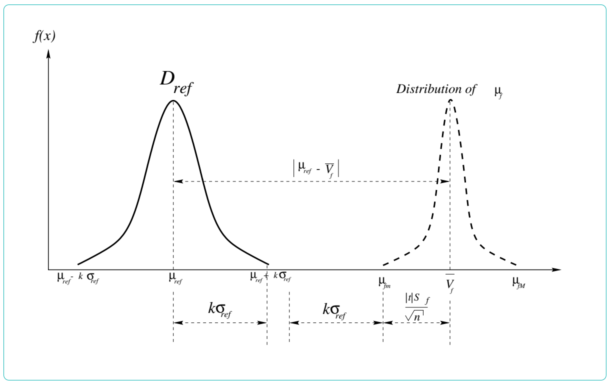

Figure 5: Computation of PDF : case $|\mu_{ref} - \overline{v}_f| \geq 2k_\alpha \sigma_{ref} + |t|\frac{S_f}{\sqrt{n}}$.

$\mu_f \in I_{\mu f} = [\mu_{fm}, \mu_{fM}]$ with $\mu_{fm} = \overline{v}_f - |t|\frac{S_f}{\sqrt{n}}$, $\mu_{fM} = \overline{v}_f + |t|\frac{S_f}{\sqrt{n}}$ and $t = t_{\alpha[n-1]}$ is the student parameter, so we can show that if

$$\boxed{|\mu_{ref} - \overline{v}_f| \geq 2k_\alpha \sigma_{ref} + |t|\frac{S_f}{\sqrt{n}} \implies PDF_f = 1} \tag{8}$$

As for the first rule, if the estimated mean $\overline{v}_f$ verifies the condition $|\mu_{ref} - \overline{v}_f| \geq 2k_\alpha \sigma_{ref} + |t|\frac{S_f}{\sqrt{n}}$, then we can stop the simulations and put $PDF_f = 1$ (see figure 5).

3.3 *Fault coverage using the PDF function*

Let's consider a set $T = \{T_1, T_2, \cdots T_n\}$ of n test patterns and a set $F = \{F_1, F_2, \cdots F_m\}$ of m faults. To get an accurate fault coverage FC of the set T, we used the PDF function as follows:

$$\boxed{FC = \frac{\sum_{j=1}^{m} Max\{PDF_{i,j} | T_i \in T\}}{m}} \tag{9}$$

where $PDF_{i,j}$ represents the probability that the test T_i detects the fault F_j and $Max\{PDF_{i,j} | T_i \in T\}$

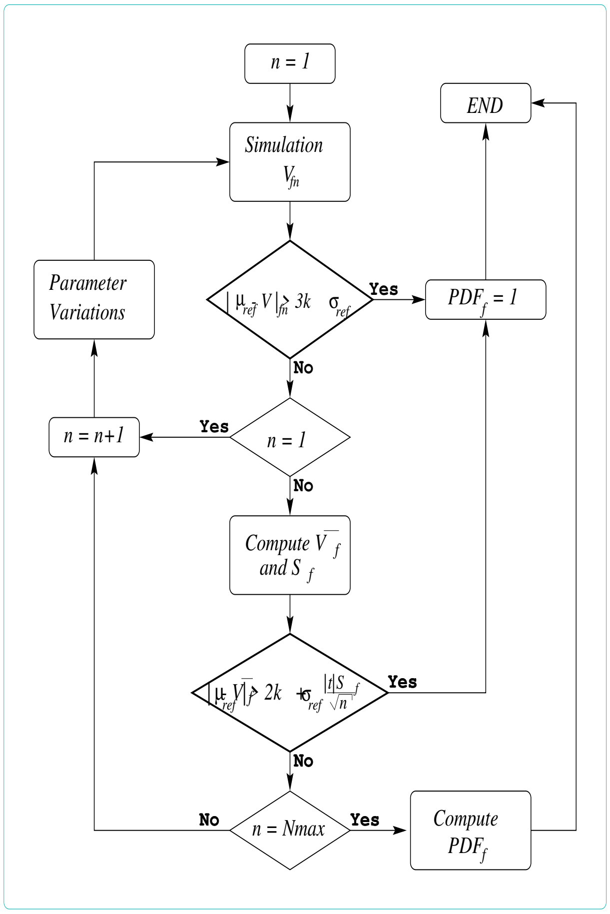

Figure 6: Fault simulation algorithm.

represents the best probability that can be obtained from all tests of the set T.

4. Analog fault simulation using the PDF function

Using the definition in (6) and the two rules corresponding to the assumptions (7) and (8), we improved our fault simulation algorithm presented in [11]. The new fault simulation algorithm is shown in figure 6.

For each faulty circuit, the algorithm starts simulations with nominal parameter values. If the simulated value v_f satisfies assumption (7) we have $PDF_f = 1$ and the simulation is stopped, else we proceed to parameter variations and re-evaluate the simulated value v_f. If the new v_f satisfies (7) then $PDF_f = 1$, else we compute the estimated $\bar{v}_f$ and s_f, if $\bar{v}_f$ satisfies assumption (8) then $PDF_f = 1$, else we repeat the above steps until the fault is detected or $n = Nmax$, where $Nmax$ represents the maximum number of simulations needed to compute the PDF value using definition (6).

5.Results

We applied our approach to the current mode integrator shown in figure 7, designed for a 0.6μ $CMOS$ process [12]. All design parameters (W and L of the transistors and capacitance values) are assigned 5% of statistical variations.

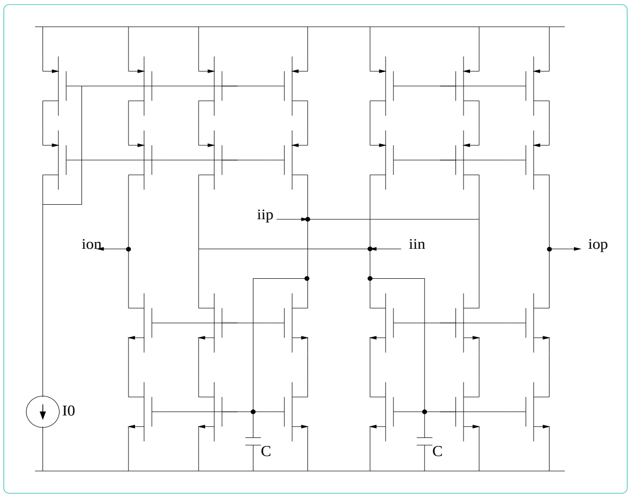

Figure 7: Current mode integrator.

type of faults	fault coverage		
	classic approach	Monte Carlo approach	our approach
catas.(54)	90.2%	85.5%	85.5%
param.(540)	84.3%	69.6%	70.4%

Table 3: Comparative fault coverage results.

We considered both catastrophic and parametric faults for transistors and capacitors. For catastrophic faults, we considered only the short faults and for parametric faults we considered 10 deviations per parameter from 10% to 100% by a step of 10%. For all transistors, we considered the occurrence of the 2 probable short faults : Gate Source Short (GSS) and Gate Drain Short (GDS) defined in [13]. So for the 26 transistors and 2 capacitors of the circuit, we have 54 catastrophic faults and 540 parametric faults. The current gain $(\frac{i_{op}-i_{on}}{i_{ip}-i_{in}})$ has been simulated using AC simulations in the frequency range $[0, 3Mhz]$.

Tables 3 and 4 give the comparison results of our approach to both the *Monte Carlo* (MC) and the classical fault simulation CS approaches considering both fault coverage and CPU time. For MC and our approach we took $Nmax = 10$ and the risk $\alpha = 0.05$. In the CS fault simulation approach, for each fault f, we make only one simulation to get the simulated value v_f of the faulty circuit, if v_f is outside the confidence interval I_{ref} of the good circuit then the fault f is detectable $PDF_f = 1$, otherwise f is undetectable $PDF_f = 0$. It can be seen from the table 3 that our method is as accurate as the MC method, while the CS method is the less accurate one. For parametric faults, the small difference in the fault

type of faults	CPU time (s)		
	classic approach	Monte Carlo approach	our approach
catas. (54)	63	632	170
param. (540)	632	6325	3426

Table 4: Comparative *CPU* time results.

coverage between MC approach ($FC = 69.6\%$) and our approach ($FC = 70.4\%$) is due to the fact, that for some parametric faults, the standard deviation σ_f of the faulty circuit is not equal to the standard deviation σ_{ref} of the good circuit. For the speed-up, we can see from the table 4 that our approach is faster than the MC method, but the speed-up is more significant for catastrophic faults (3.71 times faster) than for parametric faults (1.85 times faster). This is due to the fact that parametric faults are more difficult to detect than catastrophic faults.

6. Conclusion

We propose a new approach for accurate fault detection for analog ICs. To include the effect of deviation of parameters, the fault detection and fault coverage problems were formulated using the probability to detect a fault PDF function. We have shown the benefit of our approach on the $CMOS$ current mode integrator by computing the fault coverage of catastrophic and parametric faults with an average of 2.5 times speed-up with respect to *Monte Carlo* approach. This method is currently being studied for the implementation of a fully automated fault simulation for large analog circuits.

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