LOOP DELAY COMPENSATION IN BANDPASS CONTINUOUS-TIME ΣΔ MODULATORS WITHOUT ADDITIONAL FEEDBACK COEFFICIENTS

Hassan Aboushady and Marie-Minerve Louërat

University of Paris VI, LIP6-ASIM Laboratory,
4, Place Jussieu, 75252 Paris, France
Email: Hassan.Aboushady@lip6.fr

ABSTRACT

Loop-delay is one of the major sources of instability and Signal-to-Noise-Ratio degradation in continuous-time bandpass ΣΔ modulators. In this paper, we use the modified-z-transform technique to calculate the value of the additional feedback coefficient required to compensate for the loop-delay. It is shown that, in certain conditions, this additional feedback coefficient can be removed and the loop-delay is compensated only by modifying the modulator coefficients. This is illustrated by several examples of loop-delay compensation in 2^{nd}, 4^{th} and 6^{th} order bandpass modulators.

1. INTRODUCTION

Continuous-Time (CT) ΣΔ modulators have several advantages compared to their Discrete-Time (DT) counterparts. CT ΣΔ are theoretically capable of operating at higher sampling frequencies for low-voltage supply and with a lower power consumption than DT ΣΔ modulators.

The main drawback of CT ΣΔ modulators is their high sensitivity to any non-idealities in the feedback pulse. Loop-delay, $\frac{t_d}{T}$, is one major non-ideality that can significantly degrade the performance of CT ΣΔ modulators [1].

Loop-delay is mainly due to the comparator response-time and the latch propagation delay in the quantizer. It is also due to the propagation delay in the digital circuitry required to perform Dynamic Element Matching (DEM) of the feedback DAC elements in the case of multi-bit ΣΔ modulators.

In mono-bit ΣΔ modulators, SNR degradation due to loop-delay may be significantly reduced by using a Return-to-Zero (RZ) feedback signal [2]. In multi-bit CT ΣΔ modulators, we prefer to use Non-Return-to-Zero (NRZ) feedback signals in order to take advantage of their reduced sensitivity to clock jitter [3].

Previous work on loop-delay compensation [4], suggested to add an additional feedback signal a_x. Loop-delay is rather difficult to estimate since it is signal dependent [5], and it is also subject to process and temperature variations [1].

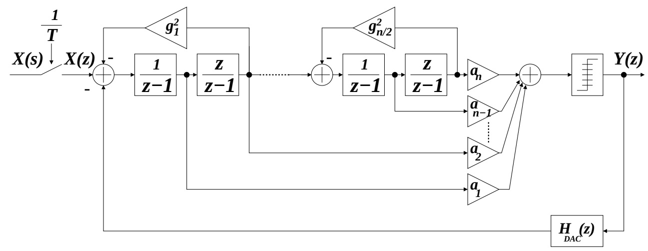

Figure 1: Discrete-time bandpass ΣΔ modulators.

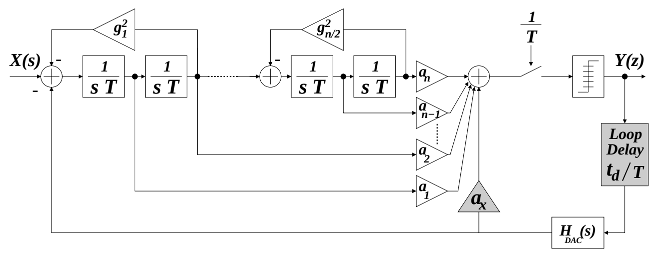

Figure 2: Continuous-time bandpass ΣΔ modulators with loop-delay, $\frac{t_d}{T}$, and feedback compensation coefficient, a_x.

In this paper, we propose to put an explicit delay of one period, $\frac{t_d}{T} = 1$, or half a period, $\frac{t_d}{T} = \frac{1}{2}$, in the feedback loop. This explicit delay should be sufficiently large to include comparator and digital circuitry delay with enough margin to include any additional delay due to signal dependency, process or temperature variations.

It will be shown that in bandpass ΣΔ modulators it is possible to compensate for the loop-delay without any additional coefficients.

2. DT-TO-CT TRANSFORMATION WITH LOOP DELAY

DT ΣΔ modulators, Figure 1, are used as a starting point to design CT ΣΔ modulators, Figure 2. This is done by comparing their respective loop gain transfer functions $G_d(z)$ and $G_c(z)$:

$$\begin{aligned} G_d(z) &\equiv G_c(z) \\ H_d(z) &\equiv \mathcal{Z}\left[\, H_{DAC}(s)\ \left(a_x + H_c(s)\right)\,\right] \end{aligned} \quad (1)$$

where $H_d(z)$, $H_c(s)$ and $H_{DAC}(s)$ are the DT loop filter, the CT loop filter and the feedback DAC transfer functions respectively.
Whenever non-idealities of the feedback pulse are involved, this DT-to-CT transformation is usually performed in the time-domain [1][4], which significantly increases the complexity of the calculations and makes it inappropriate for design automation. This is mainly due to the fact that traditional *z-tranform* techniques cannot deal with signal variations between 2 sampling instants.

In [6], a general method for DT-to-CT $\Sigma\Delta$ transformation based on the *modified-z-transform* technique was presented. It was shown that this method is valid for the different lowpass and bandpass $\Sigma\Delta$ with rectangular and non-rectangular feedback DAC signals. In this paper, we show that loop-delay, $\frac{t_d}{T}$, can be modeled using the *modified-z-transform* technique, and that this technique can also be used to calculate the loop-delay compensation coefficient, a_x.

In Figure 3(a) and 3(b), we show a NRZ feedback DAC pulse shape in the ideal case and with loop-delay, respectively. Assuming $0 \leq \frac{t_d}{T} \leq 1$, The loop gain transfer function of a CT $\Sigma\Delta$ modulator with a delayed feedback pulse can then be written in the following form:

$$G_c(z) = \mathcal{Z}\left[\frac{1-e^{-Ts}}{s}\ e^{-\frac{t_d}{T}s}\quad (a_x + H_c(s))\right] \tag{2}$$

Using the *modified-z-transform* we get:

$$G_c(z) = (1-z^{-1})\ \ \mathcal{Z}_m\left[\frac{a_x + H_c(s)}{s}\right] \tag{3}$$

where $m = 1 - \frac{td}{T}$. The *modified-z-transform* can be directly calculated from the *Laplace* representation using the residue theorem [7]. This method is systematic and convenient for design automation. Equation (3) can then be written in the following from:

$$G_c(z) = (1-z^{-1}) \sum_{p_i = \text{poles of } \frac{a_x + H_c(s)}{s}} \text{Residues of } \left.\frac{a_x + H_c(s)}{s}\frac{e^{mTs}}{z - e^{Ts}}\right|_{\text{at } p_i} \tag{4}$$

Using equation (4), we can get a general expression for the loop gain transfer function of an even order CT $\Sigma\Delta$ modulator with loop-delay:

$$G_c(z) = \frac{\alpha_{c_n} z^n + \alpha_{c_{n-1}} z^{n-1} + \cdots + \alpha_{c_1} z + \alpha_{c_0}}{z(z^2 - 2\ cos(g_{c_1})z + 1)\ldots(z^2 - 2\ cos(g_{c_{\frac{n}{2}}})z + 1)} \tag{5}$$

A similar expression of an even order DT $\Sigma\Delta$ loop gain can be described by the following relation:

$$G_d(z) = \frac{\alpha_{d_{n-1}} z^{n-1} + \alpha_{d_{n-2}} z^{n-2} + \cdots + \alpha_{d_1} z + \alpha_{d_0}}{(z^2 - (2 - g_{d_1}{}^2)z + 1)\ldots(z^2 - (2 - g_{d_{\frac{n}{2}}}{}^2)z + 1)} \tag{6}$$

From equations (5) and (6) we notice that the loop-delay has increased by one the order of both the numerator and the denominator of the CT $\Sigma\Delta$ loop gain. It is then impossible to perform equivalence between the CT and the DT loop gain transfer functions since their orders are not identical.

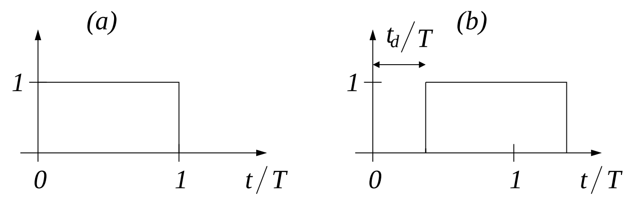

Figure 3: (a) Ideal Feedback Pulse. (b) Feedback Pulse with loop-delay $\frac{t_d}{T}$.

3. THE FEEDBACK COMPENSATION COEFFICIENT A_X

To reduce the order of the CT $\Sigma\Delta$ loop gain we will find an expression for the compensation coefficient, a_x, in function of the CT loop filter coefficients, $a_{c_1}, \ldots, a_{c_n}$ such that:

$$\alpha_{c_0}(a_{c_1}, \ldots, a_{c_n}, a_x, \frac{t_d}{T}) = 0 \tag{7}$$

Using equation (7), we can find an expression for the compensation coefficient, a_x, in function of the CT $\Sigma\Delta$ coefficients and the loop-delay:

$$a_x = f(a_{c_1}, \ldots, a_{c_n}, \frac{t_d}{T}) \tag{8}$$

By substitution from equation(8) into equation (5), we will get an expression for the CT loop gain, $G_c(z)$, having the same order as the DT loop gain, $G_d(z)$. Comparing the denominators, we can determine the local resonator feedback coefficients:

$$g_{c_i} = cos^{-1}(1 - g_{d_i}^2) \tag{9}$$

Comparing the numerators of $G_c(z)$ and $G_d(z)$ and using the same matrix representation described in [6], we can get the CT loop filter coefficients in function of the DT loop filter coefficients and the loop-delay $\frac{t_d}{T}$.

$$\begin{aligned} a_{c_1} &= f(a_{d_1}, \ldots, a_{d_n}, \frac{t_d}{T}) \\ &\vdots \\ a_{c_n} &= f(a_{d_1}, \ldots, a_{d_n}, \frac{t_d}{T}) \end{aligned} \tag{10}$$

Now that we have all the CT loop filter coefficients in function of the DT loop filter coefficients and the loop-delay, we can substitute from equation (10) into equation (8) to get the feedback compensation coefficient in function of the DT loop filter coefficients and the loop-delay:

$$a_x = f(a_{d_1}, \ldots, a_{d_n}, \frac{t_d}{T}) \tag{11}$$

All the calculations described in this section have been performed using a symbolic mathematical tool MAPLE [9]. In the following section, we will see some design examples of CT $\Sigma\Delta$ modulators with loop-delay compensation.

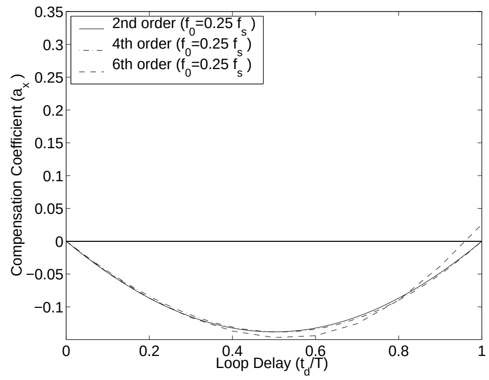

Figure 4: The feedback compensation coefficient, a_x, in function of the loop-delay, $\frac{t_d}{T}$, for bandpass $\Sigma\Delta$ modulators having $a_x = 0$ when $\frac{t_d}{T} = 1$.

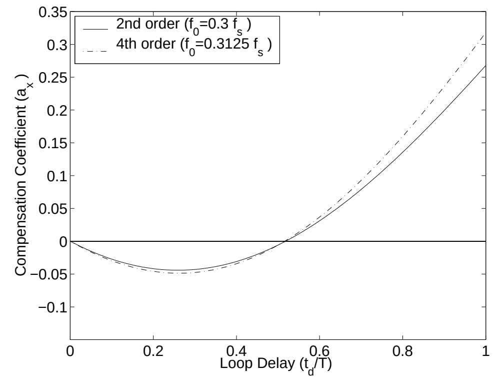

Figure 5: The feedback compensation coefficient, a_x, in function of the loop-delay, $\frac{t_d}{T}$, for bandpass $\Sigma\Delta$ modulators having $a_x = 0$ when $\frac{t_d}{T} = \frac{1}{2}$.

4. DESIGN EXAMPLES

Although the expressions developed in the previous section are valid for both lowpass and bandpass $\Sigma\Delta$ modulators, here we will only focus on examples of bandpass modulators. An expression for the compensation coefficient, a_x, in the case of a 2^{nd} order modulator can be given by:

$$a_x = -a_{d_1} sin(\frac{\pi}{2}\frac{t_d}{T}) - \frac{a_{d_2}}{2}[1 + cos(\frac{\pi}{2}\frac{t_d}{T}) + sin(\frac{\pi}{2}\frac{t_d}{T})] \quad (12)$$

Similar expressions of the CT compensation coefficient, a_x, in function of the loop-delay, $\frac{t_d}{T}$, and the DT $\Sigma\Delta$ coefficients, $a_{d_1}, \ldots, a_{d_n}$, have also been found for 4^{th} and 6^{th} order modulators. In Figure 4, we use these relations to plot the compensation coefficient in function of the loop-delay for different orders of CT bandpass $\Sigma\Delta$ modulators having their center frequency, $f_0 = 0.25 f_s$.

From Figure 4, we notice that for a loop-delay, $\frac{t_d}{T} = 1$, the value of the compensation coefficient, $a_x = 0$. This means that if we explicitly put one period delay in the feedback loop of a bandpass CT $\Sigma\Delta$ modulator, it is possible to compensate for this delay without any additional feedback coefficient. The coefficients of the DT, CT without delay and CT with one period delay modulators are listed in tables 1, 2 and 3 for the 2^{nd}, 4^{th} and 6^{th} order bandpass modulators respectively. The DT coefficients were obtained using Richard Schreier's $\Sigma\Delta$ Toolbox [8].

The compensation coefficient can also be equal to zero for other values of the center frequency, f_0. A 2^{nd} order modulator with $f_0 = 0.3 f_s$ and a 4^{th} order modulator with $f_0 = 0.3125 f_s$, have a compensation coefficient, $a_x = 0$, for a loop-delay $\frac{t_d}{T} = \frac{1}{2}$. The modulator coefficients corresponding to these cases are listed in tables 4 and 5 respectively. In order to validate the results of these calculations, we have simulated all the CT $\Sigma\Delta$ modulators presented in this section and we have compared their performances with those of the original DT modulators. The signal-to-noise ratios of the 2^{nd}, 4^{th} and 6^{th} modulators are plotted in figures 6, 7 and 8 respectively.

Table 1: Second order bandpass $\Sigma\Delta$ coeffi cients ($f_0 = 0.25 f_s$).

	DT	CT ($\frac{t_d}{T} = 0$)	CT ($\frac{t_d}{T} = 1$)
a_1	$+0.6667$	$+0.5236$	-0.5236
a_2	-0.6667	-0.8225	-0.8225
${g_1}^2$	$+2.0000$	$+2.4674$	$+2.4674$

Table 2: Fourth order bandpass $\Sigma\Delta$ coeffi cients ($f_0 = 0.25 f_s$).

	DT	CT ($\frac{t_d}{T} = 0$)	CT ($\frac{t_d}{T} = 1$)
a_1	$+0.5585$	$+0.4927$	-0.5544
a_2	-0.5585	-0.6526	-0.9922
a_3	-0.0079	-0.2777	-0.2543
a_4	-0.2083	-0.4045	$+0.4411$
${g_1}^2$	$+1.9858$	$+2.4451$	$+2.4451$
${g_2}^2$	$+2.0142$	$+2.4898$	$+2.4898$

Table 3: Sixth order bandpass $\Sigma\Delta$ coeffi cients ($f_0 = 0.25 f_s$).

	DT	CT ($\frac{t_d}{T} = 0$)	CT ($\frac{t_d}{T} = 1$)
a_1	$+0.5559$	$+0.5210$	-0.6240
a_2	-0.5559	-0.6866	-1.0640
a_3	-0.0211	-0.3417	-0.2762
a_4	-0.2219	-0.4789	$+0.6247$
a_5	-0.0433	-0.0706	$+0.1311$
a_6	$+0.0525$	$+0.1750$	$+0.1246$
${g_1}^2$	$+1.9620$	$+2.4081$	$+2.4081$
${g_2}^2$	$+2.0000$	$+2.4674$	$+2.4674$
${g_3}^2$	$+2.0380$	$+2.5275$	$+2.5275$

From these figures, we can see that there is very little difference between the performance of the original DT modulators and the performance of the CT modulators having explicit loop-delay and no compensation coefficient.

Table 4: Second order bandpass $\Sigma\Delta$ coefficients ($f_0 = 0.3f_s$).

	DT	CT ($\frac{t_d}{T} = 0$)	CT ($\frac{t_d}{T} = \frac{1}{2}$)
a_1	$+0.6048$	$+0.3339$	-0.3120
a_2	-0.8727	-1.1844	-1.2052
${g_1}^2$	$+2.6180$	$+3.5530$	$+3.5530$

Table 5: Fourth order bandpass $\Sigma\Delta$ coefficients ($f_0 = 0.3125f_s$).

	DT	CT ($\frac{t_d}{T} = 0$)	CT ($\frac{t_d}{T} = \frac{1}{2}$)
a_1	$+0.5561$	$+0.3584$	-0.3571
a_2	-0.8742	-1.1840	-1.4416
a_3	-0.0409	-0.4587	-0.4043
a_4	-0.1200	-0.3564	$+0.5564$
${g_1}^2$	$+2.7523$	$+3.8276$	$+3.8276$
${g_2}^2$	$+2.7784$	$+3.8831$	$+3.8831$

5. CONCLUSION

In this paper, we have presented a method to calculate the loop-delay compensation coefficients in CT $\Sigma\Delta$ modulators. Implementing this method, based on the modified-z-transform technique, in a symbolic mathematical tool has permitted us to study carefully the loop-delay compensation coefficient. For bandpass modulators, we have proposed to add an explicit delay of 1 or 1/2 period and to find the center frequency and the CT $\Sigma\Delta$ coefficients that would not require any compensation coefficients. Several examples of high order bandpass modulators have been given to validate the concept.

6. REFERENCES

[1] J.A. Cherry and W.M. Snelgrove. "Excess Loop Delay in Continuous-Time Delta-Sigma Modulators". *IEEE Transactions on Circuits and Systems II*, vol. 46(No. 4):376–389, April 1999.

[2] H. Aboushady, F. Montaudon, F. Paillardet, and M.M. Louërat. "A 5mW, 100kHz Bandwidth, Current-Mode Continuous-Time Sigma-Delta Modulator with 84dB Dynamic Range". *IEEE European Solid State Circuits Conference, ESSCIRC*, September 2002.

[3] R. Adams, K.Q. Nguyen, and K. Sweetland. "A 113-dB SNR Oversampling DAC with Segmented Noise-Shaped Scrambling". *IEEE Journal of Solid-State Circuits*, vol. 33(No. 12):1871–1878, December 1998.

[4] P. Benabes, M. Keramat, and R. Kielbasa. "A Methodology for designing Continuous-Time Sigma Delta Modulators". *IEEE European Design and Test Conference, EDTC*, pages 46–50, March 1997.

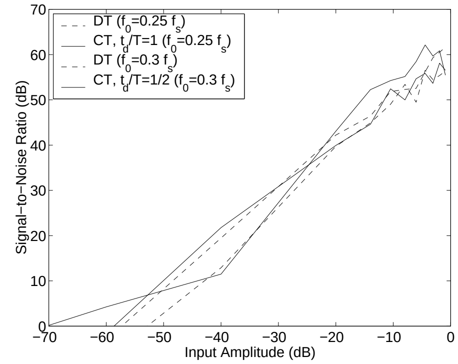

Figure 6: Second order bandpass CRFF (OSR=128).

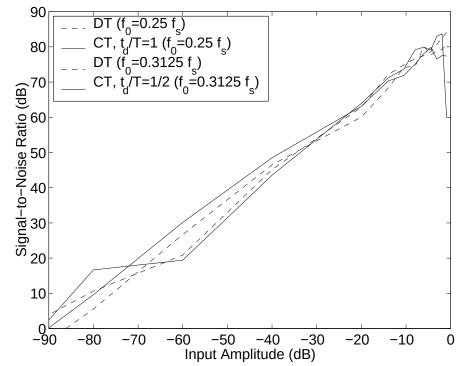

Figure 7: Fourth order bandpass CRFF (OSR=128).

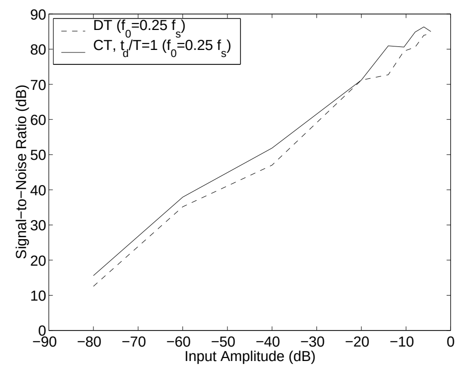

Figure 8: Sixth order bandpass CRFF (OSR=64).

[5] J.A. Cherry, W.M. Snelgrove, and P. Schvan. "Signal-Dependent Timing Jitter in Continuous-Time $\Sigma\Delta$ Modulators". *Electronics Letters*, vol. 33(No. 13):1118–1119, 19th June 1997.

[6] H. Aboushady and M.M. Louërat. "Systematic Approach for Discrete-Time to Continuous-Time Transformation of $\Sigma\Delta$ Modulators". *IEEE International Symposium on Circuits and Systems, ISCAS*, May 2002.

[7] K. Ogata. *Discrete-Time Control Systems*. Prentice-Hall International, Inc., 1987.

[8] R. Schreier. *The Delta-Sigma toolbox for MATLAB*. Oregon State University, November 1999.

[9] Waterloo Maple Inc. *MAPLE V Release 5*, 1997.