Splitting of RC-Network for Accurate Model Reduction

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Abstract

Reduction of an extracted RC-network is an important pre-processing step for techniques such as timing and crosstalk noise analysis of VLSI circuit. In a previous paper, we have described a method for reducing an RC-network to a simplified circuit. The experience shows that the accuracy of the method is satisfying but the computation time is not reasonable. In this paper, we propose to split the initial RC-network in order to decrease the computation time.

1. Introduction

As integrated circuit technologies continue to be thinner, crosstalk capacitance effects have a more dominant impact on signal integrity and timing analysis than ever before. With coupling capacitance, the transition of a wire can impact its neighbouring wires. It can cause a functional failure [1][2] or modify the timing performance of the circuit [4][5].

With deep submicron technologies (DSM), the intrinsic resistance of a wire can not be neglected. So, the wires are modelled with RC-network. In a real size circuit, the RC-network can be composed of thousand of resistance-capacitance elements and hundred of aggressors. Figure 1 shows an example of an extracted RC-network with one aggressor and 8 nodes.

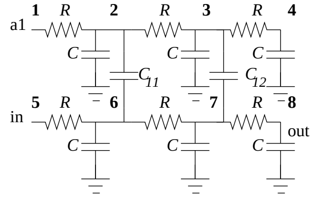

Figure 1. RC-network with 8 nodes

Several tools propose to manage RC-networks. For example, [6] and [7] propose to determine the crosstalk peak noise with RC of interconnect and [3] [10] [8] give an approximation of the delay due to the interconnect. These methods give a mathematical expression of the peak noise or of the output waveform.

Let's consider an extracted RC-network composed of $m+1$ nodes numbered of 0 to m. Note that 0 is the ground voltage, 1 to n the internal nodes and the output nodes, $n+1$ to m the input node (with a known voltage). $x_i(t)$ is the waveform of the node i in the time-domain and $x'_i(t)$ the first derivative of $x_i(t)$.

An RC-network composed of $m+1$ nodes is characterised, in the time-domain, by a system of n equations (S_t).

$$S_t \begin{cases} G_1 x_1(t) + C_1 x'_1(t) = \sum\limits_{j=0}^{m} G_{1j} x_j(t) + \sum\limits_{j=0}^{m} C_{1j} x'_j(t) \\ \vdots \\ G_n x_n(t) + C_n x'_n(t) = \sum\limits_{j=0}^{m} G_{nj} x_j(t) + \sum\limits_{j=0}^{m} C_{nj} x'_j(t) \end{cases} \tag{1}$$

where, for the line i

- G_i is the total conductance of node i: $G_i = \sum_{l=0}^{m} \frac{1}{R_{il}}$
- C_i is the total capacitance of node i: $C_i = \sum_{k=0}^{m} C_{ik}$
- G_{il} is the conductance between node i and l.
- C_{ik} is the capacitance between node i and k.

The resolution of this system gives, for each node, the following solution

$$x_i(t) = \sum_{k=1}^{n} a_{ik}.e^{-h_k.t} + a_{i0} \tag{2}$$

where a_{i0} is the voltage when $t \rightarrow +\infty$ of the node i (known value), a_{ik} a coefficient in Volt and h_k a frequency.

Our method of reduction of RC-network comprises by two steps. First, we determine the output of the RC-network according to the input. Then, we compute the simplified circuit parameters [9].

To begin, we transform the system S_t in an equivalent system S_{eq} without coupling capacitance by applying linear operations between lines.

$$S_{eq} \begin{cases} G'_1.x_1(t) + C'_1.x'_1(t) & = \sum_{j=0}^{m} G'_{1j}.x_j(t) \\ \vdots & \\ G'_n.x_n(t) + C'_n.x'_n(t) & = \sum_{j=0}^{m} G'_{nj}.x_j(t) \end{cases} \tag{3}$$

With the equivalent system S_{eq} and the initial values of each node of the circuit at time $t = 0$, we compute the first derivatives $x'_i(0)$, noted x_{i1}. Then, we compute the n first derivatives for each node at time $t = 0$, noted x_{in}. On the other hand, from the expression of the node i (eq. 2) and using derivation, we obtain a second set of equations for each node i (S_{der}) where $a_{i1}, \cdots, a_{in}$ and $h_1, \cdots, h_n$ are the variables.

$$S_{der} \begin{cases} \sum_{j=1}^{n} h_j^0 a_{ij} & = (-1)^0 x_{i0} \\ \vdots & \\ \sum_{j=1}^{n} h_j^n a_{ij} & = (-1)^n x_{in} \end{cases} \tag{4}$$

Unfortunately, S_{der} is a non-linear system. We solve this system by eliminating the variables a_{ij} by subtracting the line p and the line $p-1$. After, n iterations, we obtain the equation:

$$\begin{aligned} 0 = x_{in} + \sum_{i=1}^{n} h_i.x_{in-1} + \sum_{i=1}^{n} \sum_{j=i+1}^{n} h_i h_j.x_{in-2} \\ + \cdots + \prod_{i=1}^{n} h_i.x_{i0} \end{aligned} \tag{5}$$

We note σ_{n-1} to σ_0 such as the equation 5 can be written as

$$\sigma_{n-1} x_{in-1} + \sigma_{n-2} x_{in-2} + \cdots + \sigma_0 x_{i0} = -x_{in} \tag{6}$$

Applying this equation on each node, we obtain n equations with n unknowns.

$$\begin{cases} x_{10}\sigma_0 & + & \cdots & + & x_{1n-1}\sigma_{n-1} & = & -x_{1n} \\ \vdots & & & & & & \\ x_{n0}\sigma_0 & + & \cdots & + & x_{nn-1}\sigma_{n-1} & = & -x_{nn} \end{cases} \tag{7}$$

σ_i are obtained from the resolution of the matrix equation. Then, we determine the frequencies h_i from σ_i by computing the root of a polynomial. Now, with the frequencies h_i, we can compute the coefficients a_{ij} with the equation system S_{der}. This method gives the exact waveform of the output according to the input voltage.

With the exact waveform of the output, we determine the simplified circuit parameters. The proposed simplified circuit is composed of two parts: the victim model and the aggressor model. The simplified circuit of an RC-network composed of n aggressors is shown in figure 2. To put it more precisely, the simplified circuit can be integrated in a crosstalk or timing analysis tools and take only into account the load gate. Note that the gate of emitter is not modelled in the simplified circuit.

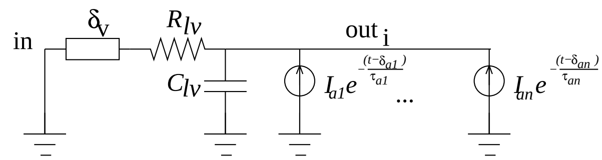

Figure 2. Simplified circuit for an RC-network composed of n aggressors

We compute the parameters of the victim (δ_v, R_{lv}, C_{lv}) by minimising the output waveform obtained with the simplified circuit (without the aggressors) and the initial RC-network when the aggressors are connected to the ground. The same technique is used to compute the parameters of each aggressors.

To resume, to compute the parameters of an RC-network composed of n aggressors, we determine the output waveform due to transition of each aggressor and without aggressors ($n+1$ iterations).

In this paper, we propose to split the initial RC-network in order to decrease the computation time. This paper is organised as follows. First, we present our method on RC-network splitting. Section 3 gives an example of use of the proposed method. Section 4 gives some results.

2. Split

Let's consider the RC-network shown in figure 3 composed of a victim coupled with two aggressors.

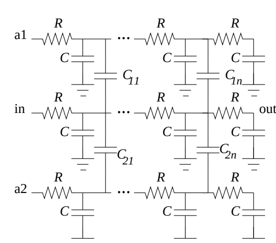

Figure 3. RC-network with two aggressors

We have seen, in the previous section, that $n+1$ iterations of the direct method are needed to compute the parameters of the simplified circuit. In addition, the complexity of the direct method is in $O(N^4)$ where N is the number of node. This means that complexity of the previous method is in $(n+1)*O(N^4)$, where n is the number of aggressors and N the number of node.

In order to decrease the complexity and the computation time of the previous method we have two solutions. We can decrease the number of aggressors or the number of node. In crosstalk or timing analysis tools, we know which aggressor is making a transition or if it is in an steady state. The steady state aggressor are disconnected. So, we can not modify the number of aggressors. Therefore, we can attempt to reduce the computation time by working on several smaller RC-networks. These networks are obtained by splitting the initial RC-network into sub-network with a smaller number of node.

Let's study the case of an RC-network composed of n aggressors and N nodes. Each aggressor has N_{a_i} nodes, the victim has N_v nodes and we have:

$$N = N_v + \sum_{i=0}^{n-1} N_{a_i} \tag{8}$$

Determining the output waveform according to the input is in $(n+1)*O(N^4)$.

We propose to use $n+1$ different circuits to compute the simplified circuit parameters. For each victim, we connect to the ground the coupling capacitance. This gives an RC-networks composed of N_v nodes. Then, for each aggressor, we disconnect all the aggressor except the one we study. The total complexity of the method is then $N_v^4 + \sum_{i=0}^{n-1} N_{a_i}^4$.

3. Example

For an RC-network composed of 2 aggressors and 150 nodes (each aggressors and the victim comprise 50 nodes), we propose to use 3 different RC-networks. First, to determine the parameters of the victim, we connect to the ground the coupling capacitance (see fig 4). We obtained an RC-network containing 50 nodes.

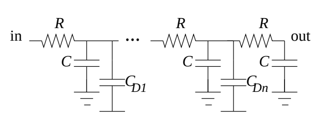

Figure 4. RC-network used to compute the victim parameters

Then, to compute the parameters of the aggressor a_1, we connect to the ground the coupling capacitances which are connected to a_2 (see fig. 5). We obtained an RC-network composed of 100 nodes.

We use the same technique to compute the parameters of the aggressors a_2 (see fig. 6) and we obtained an RC-network composed of 100 nodes.

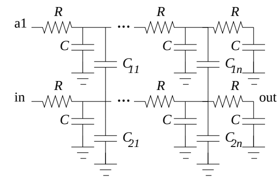

Figure 5. RC-network used to compute the parameters of the aggressor a_1

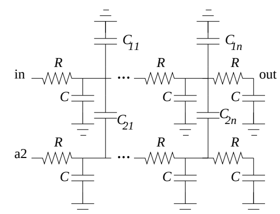

Figure 6. RC-network used to compute the parameters of the aggressor a_2

Figure 7 shows a comparison of the victim signal obtained with the

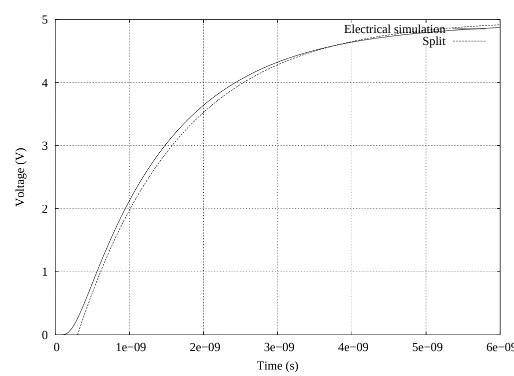

Figure 7. Output obtained with the initial circuit and the proposed method

electrical simulation of the initial circuit and the output waveform obtained with the RC-network for which all the coupling capacitance are connected to the ground. We can see that the two signals are very closed.

Figure 8 shows the signal obtained with the electrical simulation of the initial circuit and the output waveform obtained with the second RC-network (where the coupling capacitance of a_2 are connected to the ground). We can see that the peak obtained with the two curves are equal. In addition, the two curves are very closed.

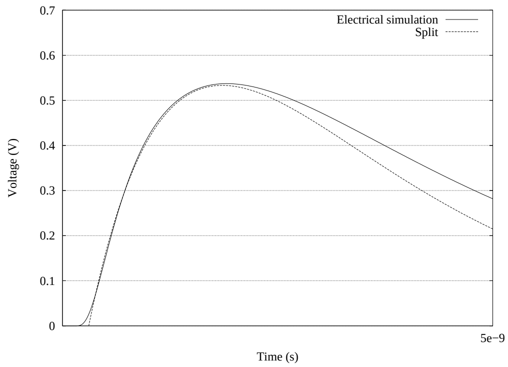

Figure 8. Output obtained with the initial circuit and the proposed method

4. Results

This method has been implemented and tested on several circuit composed of two aggressors and with a variable number of node.

The table 1 compare the peak voltage obtained without and with the splitting method compared to an electrical simulation and the computation time obtained without and with the splitting method.

	No splitting		Splitting	
Node	Error	Time (s)	Error	Time (s)
50	1.98%	1	2.18%	1
100	2.04%	1	2.20%	1
150	2.68%	4	5.89%	2
200	3.56%	15	6.95%	8
250	2.70%	37	1.77%	18
300	2.68%	77	6.86%	38
350	8.45%	142	7.64%	68

Table 1. Execution time and accuracy obtained without and with the splitting method

We can see that the computation time is divided by a factor of 2 with the splitting method. It is due to the fact that we determine the victim parameters with an RC-network composed of $\frac{N}{3}$ nodes and the aggressors parameters are computed with 2 RC-network composed of $\frac{2N}{3}$. Since the computation time grows as "number of nodes" power 4 the speed down factor is obtained by

$$(\frac{1}{3})^4 + 2.(\frac{2}{3})^4 = 0.40 \tag{9}$$

The accuracy of the splitting method is lesser than the one without the splitting method but it is still satisfying.

5. Conclusion and future works

In this article, we have proposed a method to decrease the computation time of our reduction method by splitting the initial RC-network into several smaller RC-networks. The experience shows that the use of the splitting method is twice faster than initially and the accuracy of the results are satisfying.

In the future, we will study some other techniques to decrease the computation time (for example, we can merge the node which have the same time constant). In addition, with the ultra deep submicron technologies (UDSM), other parasitic elements will appear: the inductance of interconnection. The proposed method have to be modify in order to take into account the L of the interconnection.

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