DETERMINING THE ANALYTIC WAVEFORM OF AN RC-CIRCUIT OUTPUT

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ABSTRACT: In very deep submicron technologies, the parasitic capacitor and resistance can have a significant impact on propagation delays. The Elmore delay metric is widely used due to its efficiency and ease of use. However, it is well know that this method can have significant error on large RC-circuit. In this paper, we present a new method for determining the analytic waveform of the RC-circuit outputs. The accuracy of this method is demonstrated on several large RC interconnect circuits.

INTRODUCTION

Multi-million transistor circuits are made using the latest processes. The features of these technologies include an increased number of metal levels, thinner metal width, increased wire height versus width ratio and smaller wire spacing. These new features introduce new cause of failure. This is the reason why designers spend up to 80% of a design on the verification step. Therefore, some new verification tools are needed to check the robustness of VLSI circuits against these causes within a reasonable computation time.
It is well known that some up to lately neglected physical effects in submicron technologies, such as parasitic capacitor and resistance, can significantly affect the behaviour of the circuit (timing and/or functional failure). Nowadays, the design methodologies [1] and tools, such as router [2] and verification tools, have to take into account these parasitic elements [3] [4] [5].
In real-size circuit, the RC-circuits can have a really important number of parasitic elements (near 1000 or 10000 resistances and ground capacitors) and have a complex topology with crosstalk coupling capacitor. Thus, the parasitic RC-circuits can not be directly taken into account in verification tools. We propose to reduce the complexity of the RC-circuit by modeling the RC-circuits with a simplified circuit wich can be used in verification tools. The method is composed of two steps. First, we compute the output waveform according to the input waveform. Then, we determine the parameters of the simplified model. In this article we expose the method used to compute the output waveforms.
This paper describes an original approach for determining the analytic waveform of the RC-circuit outputs and is organised as follow. We describe the two main methods used to reduce the RC-tree. Then, in section 3 we give a mathematical formulation of the problem. In section 4, our method is developed, based on the frequency domain. Section 5 shows some results. Finally, some conclusions and future works are offered in section 6.

PREVIOUS WORKS

The two main methods used to reduce RC-circuit are the Elmore delay metric and the first three moments method.

Elmore

The Elmore delay metric [6] is today widely used in current physical design tool. The popularity of the metric is mainly due to its efficiency and ease of use.
Elmore defined the 50% propagation delay at a given node i as

$$x_i(T_{Di}) = \frac{V_{DD}}{2} \tag{1}$$

where x_i is the voltage of node i.
T_{Di} can easily be expressed when the RC-circuit is a tree [7]. In this case, the 50% propagation delay at node i is

$$T_{Di} = \sum_{k \in \delta} R_{ki} C_k \tag{2}$$

where δ is the set of internal nodes, R_{ki} is the common resistance from input to the nodes i and k and C_k is the capacitor at node k.
Let's consider the RC-circuit of the figure 1. in is the input signal and out the output signal.

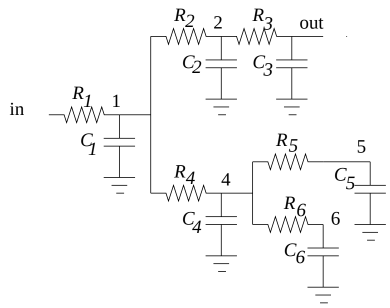


Fig. 1. Example of RC-circuit

The Elmore delay for the output signal is

$$\begin{aligned} T_{D_{out}} = & C_1 R_1 + C_2(R_1 + R_2) + C_3(R_1 + R_2 + R_3) \\ & + C_4 R_1 + C_5 R_1 + C_6 R_1 \end{aligned} \tag{3}$$

In order to compare the Elmore method with an electrical simulation, we built a function f_{Elmore} such as

$$f_{Elmore}(t) = V_{DD}(1 - e^{-\frac{t}{\tau}}) \tag{4}$$

and we determine the time constant τ such as

$$f_{Elmore}(T_{D_{out}}) = \frac{V_{DD}}{2} \tag{5}$$

Figure 2 shows the electrical simulation and the function f_{Elmore} obtained with the Elmore method.

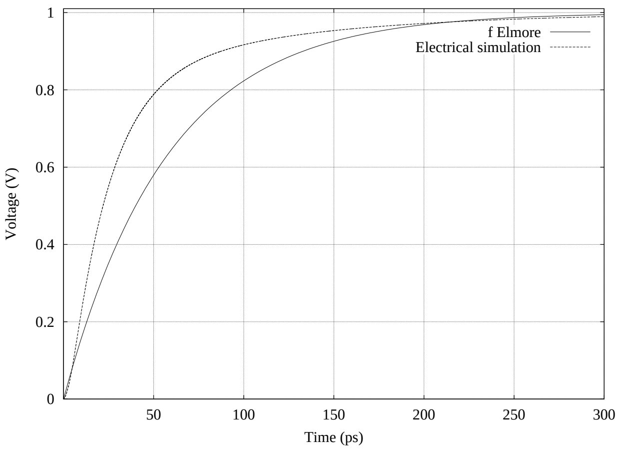


Fig. 2. Electrical simulation and Elmore method

With the Elmore method, it is easy to determine the 50% propagation delay of an RC-tree. However, if the RC-circuit has some coupling capacitor, the expression of the time T_{De} can not be expresssed easily. In addition, even for a tree, this method can be inaccurate.

First Three Moments Method

Recently, the first three moments method [8] gives the explicit expression for the delay as a function of the first three moments of the impulse response.
This method is based on the transfert function, H_{out}, of the RC-circuit in the frequency domain, defined as

$$H_{out}(s) = \frac{X_{out}(s)}{X_{in}(s)} \tag{6}$$

where X_{in} is the input signal and X_{out} the output signal. The output signal is then approximated as

$$\hat{x}_{out}(t) = k_1.e^{p_1.t} + k_2.e^{p_2.t} \tag{7}$$

where k_1, k_2, p_1 and p_2
In the frequency domain the same expression can be written as

$$\hat{H}_{out}(s) = \frac{k_1}{s - p_1} + \frac{k_2}{s - p_2} \tag{8}$$

A series expansion near s=0 gives

$$\hat{H}_{out}(s) = -(\frac{k_1}{p_1} + \frac{k_2}{p_2}) - (\frac{k_1}{p_1^2} + \frac{k_2}{p_2^2}).s - (\frac{k_1}{p_1^3} + \frac{k_2}{p_2^3}).s^2 - (\frac{k_1}{p_1^4} + \frac{k_2}{p_2^4}).s^3 - \cdots \tag{9}$$

where $p_1 < p_2 < p_3 < \cdots < p_n$
On the other hand, for each node i of the RC-circuit, the expression of the node $\tilde{H}_i$ can be determined in the frequency domain, with a series expansion around s=0 and written as

$$H_i(s) = m_0 + m_1.s + m_2.s^2 + m_3.s^3 + ... \tag{10}$$

where m_i is the i^{th} moment.
So, we can have the output function

$$H_{out}(s) = m_0 + m_1.s + m_2.s^2 + m_3.s^3 + ... \tag{11}$$

Then, p_1, p_2, k_1 and k_2 can be obtained by matching m_0, m_1, m_2 and m_3 with the coefficients of $\hat{H}_{out}(s)$.

$$p_1 = \frac{m_2}{m_3} \qquad p_2 = p_1.\frac{\frac{m_0}{m_1} - \frac{m_1}{m_2}}{\frac{m_1}{m_2} - \frac{m_2}{m_3}}$$

$$k_1 = \frac{1 + m_1 p_2}{p_1 - p_2} p_1^2 \qquad k_2 = \frac{1 + m_1 p_1}{p_1 - p_2} p_2^2 \tag{12}$$

Let's study an RC-circuit composed by two coupling capacitors 3. We suppose that the node in is in a steady-

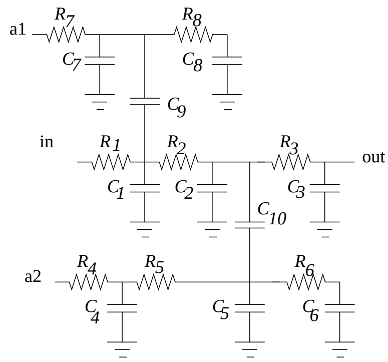


Fig. 3. RC-circuit with two coupling capacitors

state and that the aggressors a_1 and a_2 are making a transition from V_{SS} to V_{DD}. Figure 4 shows an electrical simulation and the result obtanied with the first three moments method.

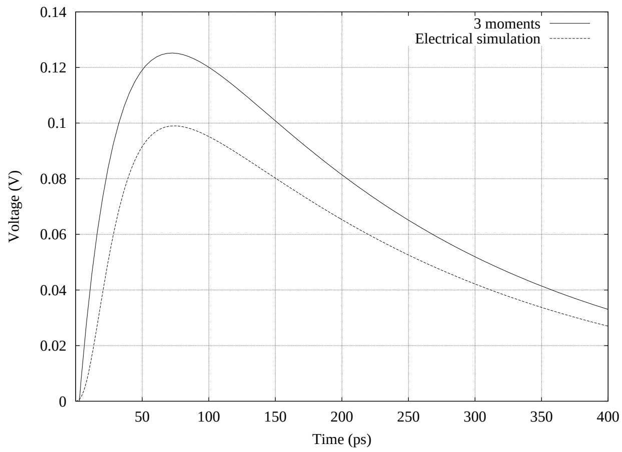


Fig. 4. Electrical simulation and first three moments method

We can see that the electrical simulation and the first three moments method are not very closed. This is mainly due to the series expansion. Indeed, without coupling capacitor, an RC-tree is a low pass filter. That explains the use of series expansion near s=0. The coupling capacitors are the high pass filters and generate the inaccuracy of this method.

PROBLEM FORMULATION

Let's study a wire (V), called victim, coupled with several thousand of wire (A_i) called aggressors. Each agressors can be coupled with thousands of wire called secondary victims. In deep submicron technologies, this set of wire is modeled by a RC-circuit.

Now, let's consider an RC-circuit composed by $m+1$ nodes numbered of 0 to m. We note $X_i(s)$ the ith-node waveform in the frequency domain. We suppose 0 is the ground voltage and $X_0(s) = 0$. Some nodes, called input nodes, represent the initial point of a wire. These nodes, connected to output gates, have a known voltage. Some node, called output nodes, represent the final point of the wire. They are connected to input gates. Except for input and output nodes, there are internal nodes, which are not connected to gates.

Let's consider a node i connected to the node l through a resistor R_{il} and to the node k through a capacitor C_{ik}. C_{i0} is the ground capacitor (see figure 5).

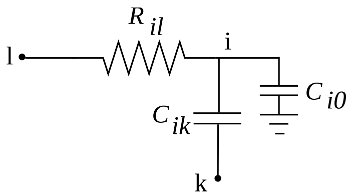


Fig. 5. Node i connected to several nodes with a resistor and a capacitor

We have the equation:

$$\sum_{j=0}^{m} \frac{x_i - x_j}{R_{ij}} + \sum_{j=0}^{m} C_{ij}(x_i' - x_j') = 0 \qquad (13)$$

Which can be written as:

$$G_i.x_i + C_i.x_i' = \sum_{j=0}^{m} G_{ij}.x_j + \sum_{j=0}^{m} C_{ij}.x_j' \qquad (14)$$

where

- G_i is the total conductance of node i: $G_i = \sum_{j=0}^{m} \frac{1}{R_{ij}}$
- C_i is the total capacitance of node i: $C_i = \sum_{j=0}^{m} C_{ij}$
- G_{ij} is the conductance between node i and j: $G_{ij} = \frac{1}{R_{ij}}$. When the nodes i and j are not connected by a resistor, $G_{ij} = 0$ (particularly $G_{ii} = 0$).
- C_{ij} is the capacitance betweenœuds i et j. When the nodes i and j are not connected by a capacitor, $C_{ij} = 0$ (particularly $C_{ii} = 0$)

We note 0 the ground voltage. 1 to l are the internal nodes, $l+1$ to n the output nodes and $n+1$ to m the input nodes.

With the equation 14, a RC-circuit composed by $m+1$ nodes is characterized, in the frequency domain, by a system of n equations (S_l).

$$\begin{cases} G_1X_1(s) + sC_1X_1(s) = \sum_{j=0}^{m} G_{1j}X_j(s) + \sum_{j=0}^{m} sC_{1j}X_j(s) \\ \vdots \\ G_iX_i(s) + sC_iX_i(s) = \sum_{j=0}^{m} G_{ij}X_j(s) + \sum_{j=0}^{m} sC_{ij}X_j(s) \\ \vdots \\ G_nX_n(s) + sC_nX_n(s) = \sum_{j=0}^{m} G_{nj}X_j(s) + \sum_{j=0}^{m} sC_{nj}X_j(s) \end{cases} \qquad (15)$$

Note that for each i, j $C_{ij} = C_{ji}$ and $G_{ij} = G_{ji}$.

We know that this system gives, for each node, the following differential equation

$$\alpha_n s^n X_i(s) + \alpha_{n-1} s^{n-1} X_i(s) + \cdots + \alpha_0 s^0 X_i(s) = 0 \qquad (16)$$

and the solution is

$$X_i(s) = \sum_{k=1}^{n} \frac{a_{ik}}{s + h_k} \qquad (17)$$

It gives in time-domain

$$x_i(t) = \sum_{k=1}^{n} a_{ik}.e^{-h_k.t} + a_{i0} \qquad (18)$$

where a_{i0} is the voltage when $t \to +\infty$ of the node i, a_{ik} a coefficient in Volt and h_k a frequency.

DETERMINING THE ANALYTIC WAVEFORM

In order to determine the n coefficients and the n frequencies, the method is composed by five steps. First, the system can be written with the following matrix equation

$$M.X(s) = 0 \qquad (19)$$

where M is

$$\begin{pmatrix} G_1+sC_1 & \cdots & -G_{1l}-sC_{1l} & \cdots & -G_{1n}-sC_{1n} & G_{1n+1} & \cdots & -G_{1m} \\ \vdots & \cdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -G_{l1}-sC_{l1} & \cdots & G_l+sC_l & \cdots & -G_{ln}-sC_{ln} & G_{ln+1} & \cdots & -G_{lm} \\ \vdots & \cdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -G_{n1}-sC_{n1} & \cdots & -G_{nl}-sC_{nl} & \cdots & G_n+sC_n & G_{nn+1} & \cdots & -G_{nm} \\ 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \end{pmatrix} \qquad (20)$$

and

$$X = \begin{pmatrix} X_1(s) \\ \vdots \\ X_l(s) \\ \vdots \\ X_n(s) \\ X_{n+1}(s) \\ \vdots \\ X_m(s) \end{pmatrix} \tag{21}$$

Then, $n-1$ iterations of Gauss-Jordan elimination method give the output waveform according to the input waveforms.

$$M_{Gauss}.X_{l+1,m}(s) = 0 \tag{22}$$

where M_{Gauss} is

$$\begin{pmatrix} P_{l+1l+1} & 0 & \cdots & \cdots & 0 & P_{l+1n+1} & \cdots & P_{l+1m} \\ 0 & P_{l+2l+2} & \ddots & \cdots & 0 & P_{l+2n+1} & \cdots & P_{l+2m} \\ \vdots & 0 & \ddots & \ddots & \vdots & \vdots & \cdots & \vdots \\ \vdots & & 0 & P_{n-1n-1} & 0 & P_{n-1n+1} & \cdots & P_{n-1m} \\ & \cdots & & 0 & P_{nn} & P_{nn+1} & \cdots & P_{nm} \\ 0 & \cdots & & 0 & \cdots & & & 0 \\ \vdots & & & & & & & \vdots \\ 0 & \cdots & & 0 & \cdots & & & 0 \end{pmatrix} \tag{23}$$

where

$$X_{l+1,m}(s) = \begin{pmatrix} X_{l+1}(s) \\ \vdots \\ X_n(s) \\ X_{n+1}(s) \\ \vdots \\ X_m(s) \end{pmatrix} \tag{24}$$

So, for each output $o \in [l+1; n]$, we obtain

$$P_{oo}(s)X_o(s) = \sum_{i=m+1}^{n} P_i(s).X_i(s) \tag{25}$$

where $P_i(s) = \sum_{j=1}^{u} p_{ij}.s^j$ and $P_{oo}(s) = \sum_{j=1}^{v} p_{oo}.s^j$.
Note that $deg(P_{oo}) < deg(P_i)$. In the initial matrix, the polynomials of the column $n+1$ to m are polynomials of degree 0. If the polynomials of column 1 to n of the initial matrix have a degree of 1, the polynomial P_{nn} of the final matrix has a degree of 2^{n-1}. Indeed, the first iteration of Gauss-Jordan elimination multiplies by 2 the degree of polynomials. $n-1$ iteration of Gauss-Jordan elimination are needed to obtained P_{nn}. So the degree of P_{nn} is 2^{n-1}.
We have seen in problem formulation section that the output waveform of a RC-circuit composed by m nodes has exactly n coefficients and frequencies. Thus, the polynomial $P_{oo}(s)$ divided by the polynomial $P_i(s)$ must have a degree of n. This means that some roots of $P_{oo}(s)$ are also roots of $P_i(s)$. In order to simplify the equation, we determine the greatest common divisor (GCD) between $P_i(s)$ and $P_{oo}(s)$ using the Euclidean algorithm.

$$\frac{P_i(s)}{P_{oo}(s)} = \frac{GCD(s).Q_i(s)}{GCD(s).Q_{oo}(s)} \tag{26}$$

with $Q_{oo}(s) = \sum_{j=1}^{n} q_{oo}.s^j$
So, we have

$$X_o(s) = \sum_{i=m+1}^{n} \frac{Q_i(s)}{Q_{oo}(s)}.X_i(s) \tag{27}$$

Now, considering only one input i gives

$$X_{outi}(s) = \frac{Q_i(s)}{Q_{oo}(s)}.X_i(s) \tag{28}$$

X_{outi} can be rewritten using partial fraction decomposition

$$X_{outi}(s) = \sum_{j=1}^{n} \frac{b_{ij}}{s+h_j} X_i(s) \tag{29}$$

where $h_j = -r_j$, r_j are the root of Q_{oo} and

$$b_{ij} = \frac{Q_i(r_j)}{\prod_{\substack{k=1 \\ k \neq j}}^{n} r_k - r_j} \tag{30}$$

The roots are computed with the Newton-Raphson method. Substitute this result into 27 to give

$$X_{out}(s) = \sum_{i=m+1}^{n} \sum_{j=1}^{n} \frac{b_{ij}}{s+h_j} X_i(s) \tag{31}$$

When the input makes a transition we have

$$X_{out}(s) = \sum_{i=m+1}^{n} \sum_{j=1}^{n} \frac{b_{ij}}{s+h_j} \frac{1}{s}.x_i \tag{32}$$

The inverse Laplace gives

$$x_{out}(t) - x_{out}(0) = \int_0^t \Big(\sum_{i=m+1}^{n} \sum_{j=1}^{n} b_{ij} e^{-h_j.t_0}.x_i \Big).dt_0 \tag{33}$$

We suppose the node out is in the steady-state V_{SS} at $t=0$.

$$x_{out}(t) = -\sum_{i=m+1}^{n} \sum_{j=1}^{n} \frac{b_{ij}}{h_j} (e^{-h_j.t} - 1).x_i \tag{34}$$

The experience has shown that the use of traditional float (64 bits) is not appropriate because the range of the polynomial coefficient is very important. We have used the GNU Multiple Precision Arithmetic Library [9] to represent the floats with a variable number of bits.
The proposed method gives the numerical formulas of the output signals. The accuracy of this method depends mainly on the

- the number of bits used to represent the coefficients
- the precision of the root finding method

RESULTS

A prototype tool that implements all the concepts described in this article has been developed. As shown in figure 6, the prototype is composed by five steps : determine the matrix equation, calculate the Gauss-Jordan elimination, find the polynomial roots, modify the equation by using the partial fraction decomposition and make the inverse Laplace.

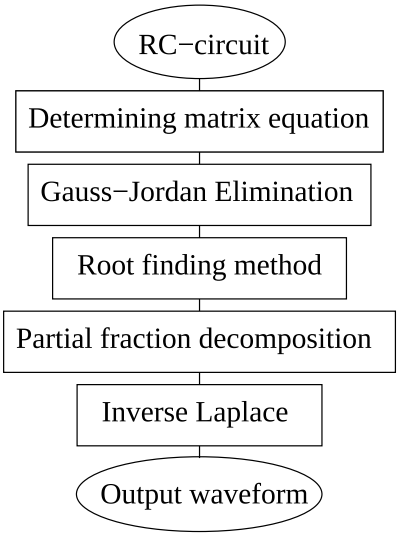
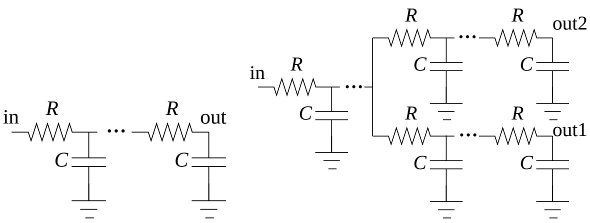
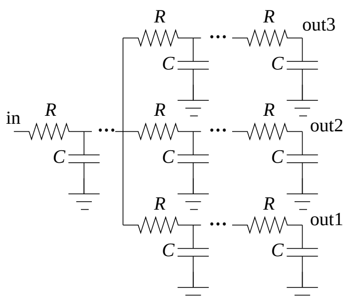


Fig. 6. Prototype tool

This prototype has been tested over three kind of RC-circuits : trees with one, two or three outputs and a variable number of internal nodes ranging from 100 to 1000 (see fig. 7).



(a) Tree with one output (b) Tree with two outputs



(c) Tree with two outputs

Fig. 7. Benches

The results presented in this section are obtained on a 1 GHz PC computer with 1 GByte RAM.

Number Of Bits Used To Represent Floats

Figure 8 gives the number of bits used to represent floats for each type of tree and different number of internal node.

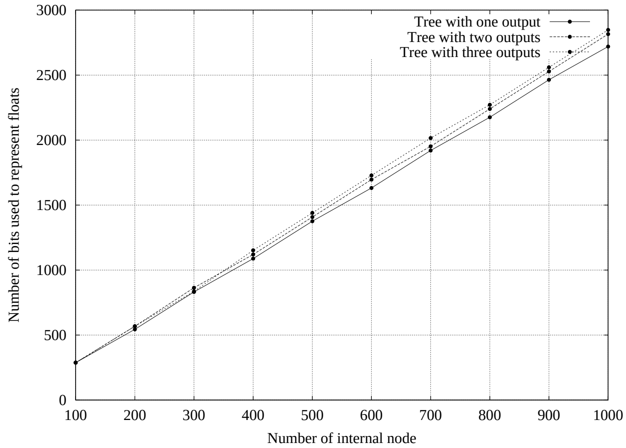


Fig. 8. Number of bits used to represents floats

We can see that the number of bits used to represent the floats is globally linear with the number of internal node. In addition, the trees with three outputs are more expensive in bits than the trees with two outputs. For example, near 2700 bits are needed to represent floats for a tree with three outputs and 1000 nodes.

Execution Time

The figure 9 shows the execution time obtained to determine the frequencies h_j. We can see that 25000 seconds

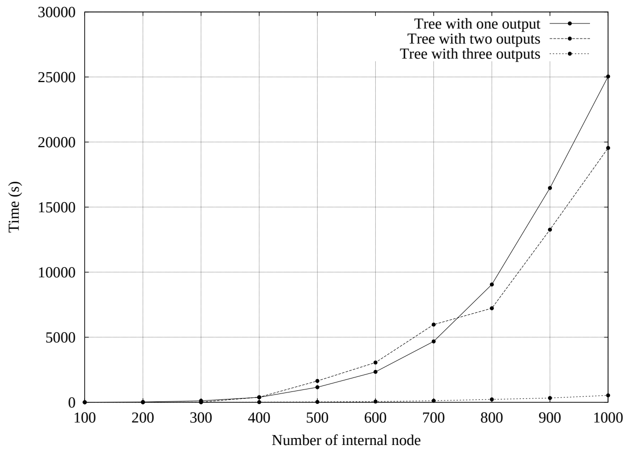


Fig. 9. Execution time for determining the frequencies

are needed to compute the frequencies of a tree with one output and 1000 internal nodes. This time execution is not appropriate for verification tools.
To determine the constant times we used a Gauss-Jordan elimination apply on polynomials (N^3). In addition, the number of bits used to represent floats is linear with the number of internal node. Thus, the complexity to com-

pute the frequencies is N^4 where N is the number of internal node.
Figure 10 shows the execution time obtained to compute the coefficients a_{ij}. We can see that for a tree with one output, 70s are needed to compute the coefficients. A

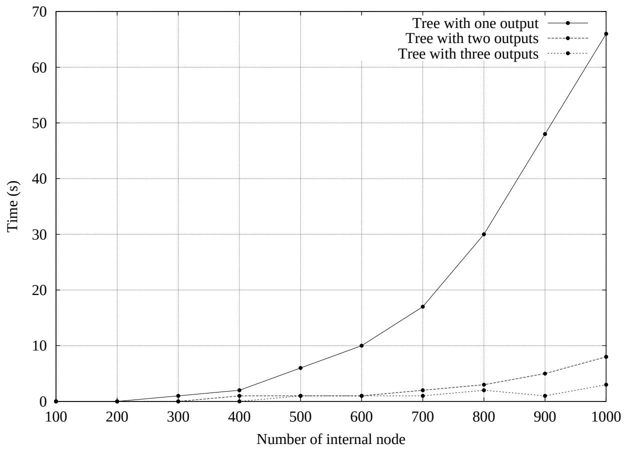


Fig. 10. Execution time for determining the coefficients

partial fraction decomposition is used to determine the coefficient. This gives a complexity in N^3.

CONCLUSION

Signal integrity is becoming a major issue in the verification process of high performance designs. Coupling capacitance and RC-circuit of interconnect are one of the factors that may cause timing and functional failure in the circuit.
In this paper, we have presented a method for determining the analytic waveform of an RC-circuit output without approximation, based on representation in the frequency domain. The precision is exact but the computation time is really important compared to an electrical simulation. On the other hand, the electrical simulation gives numerical points whereas this method gives numerical functions. The proposed method can be improved following two directions. First, we can developed techniques to accelerate the method in decreasing the number of internal node. We can analyse the RC-circuit and merge the node which have the same constant time or we can split a RC-circuit in several RC-circuits. Thus, the analytic waveform of each outputs is calculated more quickly. The second improvement concerns the method. For very deep submicron processes the inductance of the interconnect can have a significant impact on the circuit. Thus, the method has to be modified to take into consideration the inductance of the interconnect.

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