A Simplified Circuit to Model RC Interconnect

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Abstract: In very deep submicron technologies, the parasitic capacitor and resistance can have a significant impact on propagation delay and functional failure. Several methods consist in evaluating the output delay or giving an approximation of the output signal. These methods are really simple and are easily used in timing analysis. However, they are unusable in functional failure analysis such as crosstalk noise analysis.
In this paper we propose to model rc-circuits with a simplified circuit composed by a resistor, a capacitor and several current sources. The simplified circuit can easily be integrated in a crosstalk noise evaluation tool. Its accuracy is demonstrated on several large rc-circuits.

Keywords: RC-Interconnect, Modelling, Verification Tool.

1 Introduction

Multi-million transistor circuits are made using the latest processes. The features of these technologies include an increased number of metal levels, thinner metal width, increased wire height versus width ratio and smaller wire spacing. These new features introduce new cause of failure. This is the reason why designers spend up to 80 % of a design on the verification step. Therefore, some new verification tools are needed to check the robustness of VLSI circuits against these causes within a reasonable computation time.

It is well known that some up to lately neglected physical effects in submicron technologies, such as parasitic capacitor and resistance, can significantly affect the behaviour of the circuit (timing and/or functional failure). Nowadays, the design methodologies [1] and tool, such as router [2], have to take into account these parasitic elements [3].

In real-size circuit, the RC-circuits can have a really important number of parasitic elements (near 1000 or 10000 resistances and ground capacitors) and have a complex topology with crosstalk coupling capacitor. Thus, the parasitic RC-circuits can not be integrated into verification tools. We propose to reduce the complexity of the RC-circuit in modeling the RC-circuits with a simplified circuit wich can be integrated into verification tools. The method is composed by two step. First, we compute the output waveform according to the input waveform. Then, we determine the parameters of the simplified model. In this article we expose the method used to compute the output waveforms according to the input waveform.

The two main methods used to reduce rc interconnect trees are the Elmore delay metric [4] and the first three moments method [5]. Despite it being more than 50 years old, the Elmore delay metric is today widely used in current physical design tool. The popularity of the metric is mainly due to its efficiency and ease of use. Recently, the first three moments method gives the explicit expression for the delay as a function of the first three moments of the impulse response. These methods give an approximation of the delay due to interconnect. However, this delay can not be used to determine the crosstalk noise.

The crosstalk noise is the noise induce on a steady state signal called victime by its neigbouring wire called aggressors. The active aggressors are the aggressors, which are making a transition, and the silent aggressors the aggressors, which are in a steady state. In addition, the secondary victims are the signals coupled with the aggressors.

Many models have been proposed to estimate the peak value of the noise produced on a victim [6] [3] [7]. Some of them take into account the resistance-capacitance of the interconnect, others focus on studying the noise produced by the simultaneous transition of several aggressors.

Now, we expose our model that ignores the rc of the interconnect but gives a satisfying estimation of the peak produced on the victim by several aggressors using a simple approach [8]. Note that we called victim the studied signal and aggressors the signals coupled with the victim. The active aggressors are the aggressors, which are making a transition, and the silent aggressors the aggressors, which are in a steady state. In addition, the secondary victims are the signals coupled with the aggressors.This model takes into consideration some second order effects such as the existence of silent aggressors and secondary victims and is composed by three successive approximations:

- replacing signal's drivers by a simple resistance
- replacing silent aggressors and secondary victims by equivalent capacitors
- replacing active aggressors by an equivalent current

source $i_k(t) = I_{0_k} e^{-t/\tau_{i_k}}$

Then, the final equivalent circuit of a victim coupled with n aggressors is shown in figure 1.

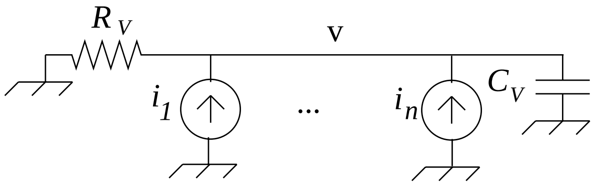

Fig. 1: Final equivalent circuit

And the victim's waveform is expressed as

$$x_v(t) = R_v \sum_{k=1}^{n} I_{0_k} \frac{\tau_{ik}}{\tau_v - \tau_{ik}}.(e^{-t/\tau_v} - e^{-t/\tau_{i_k}}) \qquad (1)$$

where $\tau_v = R_v C_v$
As we have seen, for each victim, determining which of its aggressors may commute in the same time is a key point in the noise analysis. With the input instability period and the propagation delay through the gates, we determined the noise configuration [9]. A noise configuration is defined as a subset of aggressors that may commute in the same time. An event-driven approach, based on the evaluation of instability periods, is used to make distinction between active and silent aggressors.
The experience shows that the error compared to an electrical simulation remains less than 15%. However, this method doesn't take into account the rc-circuit of interconnection.
In this paper, we propose to model the rc-circuits with coupling capacitance in order to add them in the crosstalk noise evaluation tool. The next section presents the simplified circuit used to represent the victim and aggressors. In the section 3, we show how this model can be integrated to the noise evaluation tool. Section 4 details the method used to calculated the parameters of the simplified circuit. Some results are shown in section 5. The last section exposes the concluding remarks and the future works.

2 Simplified circuit

2.1 Victim model

Let's consider a simple rc-circuit composed by six resistors and capacitors (Fig 2).

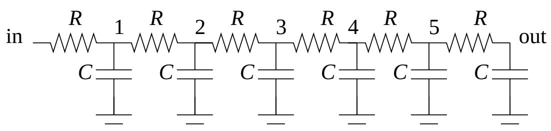

Fig. 2: RC-circuit with six nodes

When the input signal makes a transition, we know that the output waveform is:

$$x_{out}(t) = \sum_{i=1}^{6} a_i.e^{-\frac{t}{\tau_i}} + V_{DD} \qquad (2)$$

where a_i are called the coefficients and τ_i the time constant. The figure 3 shows the electrical simulation of the rc-circuit with six nodes when the resistors are equal to 5Ω and the capacitors to $1pF$ and $2pF$. We can see that the capacitor values modify the signal slope and the time where the signal is near zero.

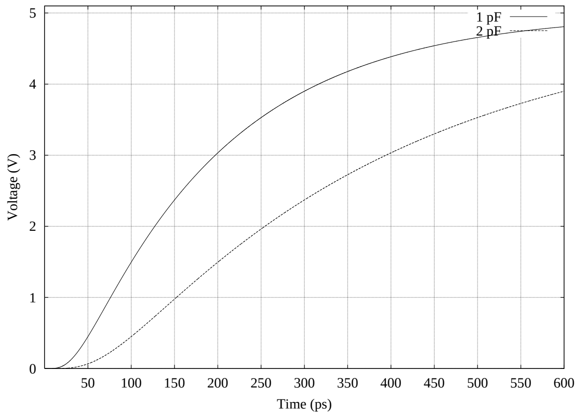

Fig. 3: Electrical simulation

We propose to model the output signal with a delay δ_v and an exponential function. The exponential function is obtained with a resistor R_{lv} and a capacitor C_{lv}. The victim model is represented in figure 4.

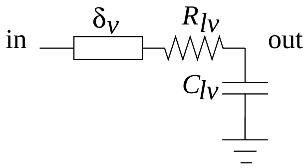

Fig. 4: Victim model

Figure 5 shows an electrical simulation of the victim model.

When the input signal makes a transition from V_{SS} to V_{DD} the output signal can be expressed as

$$\hat{x}_{out}(t) = \left| \begin{array}{ll} 0 & t \in [0, \delta_v] \\ V_{DD}.(1 - e^{-\frac{(t-\delta_v)}{\tau_v}}) & t \in [\delta_v, +\infty[\end{array} \right. \qquad (3)$$

2.2 Aggressor model

In [8] the active aggressors a_i are replaced by an equivalent current source defined as $I_{ai} e^{-\frac{t}{\tau_{a_i}}}$ where I_{ai} is the current maximum and τ_{a_i} the time constant. This approximation is really simple and the current generated by several

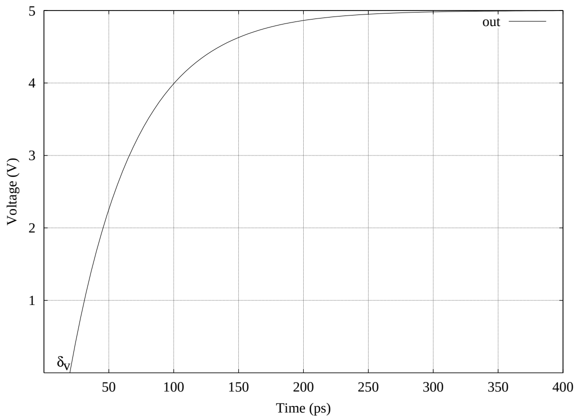

Fig. 5: Output signal of the simplified circuit

aggressors can be added. We have chosen to use a similar approximation. However, in order to take into account the rc-circuit, we have to modify this model.
Let's consider a victim composed by n nodes coupled with an aggressor (see figure 6).

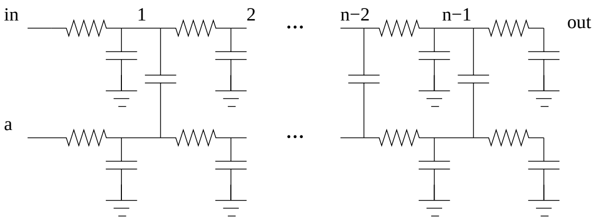

Fig. 6: Victim coupled with an aggressor

Figure 7 shows the electrical simulation of a steady-state victim composed by 3 and 6 nodes coupled with an aggres-

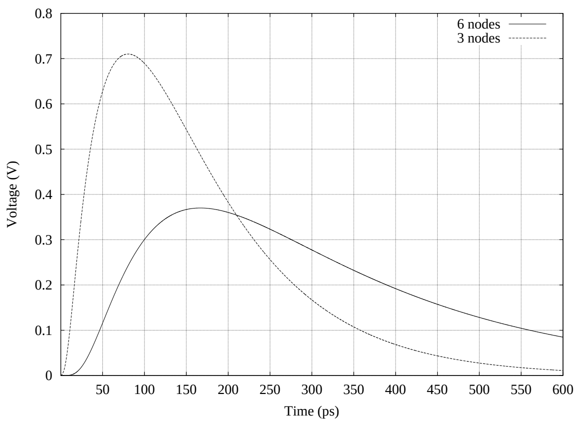

Fig. 7: Crosstalk noise due to the transition of the aggressor

sor which makes a transition from V_{SS} to V_{DD}.
We can see that the noise is significant after some delay. Thus, we propose to add a delay δ_a to the current source

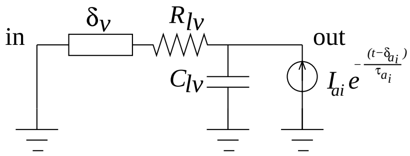

Fig. 8: Simplified circuit for a rc-circuit composed by one aggressor

describe in [8]. The current source is $I_{ai}e^{-\frac{t-\delta_{a_i}}{\tau_{a_i}}}$.
Figure 8 shows the simplified circuit of a victim coupled with one aggressor.
The output signal can be expressed as

$$\hat{x}_{out}(t) = \left| \begin{array}{ll} 0 & t \in [0, \delta_a] \\ -\dfrac{I_a \tau_a}{C_{lv}}.(1 - e^{-\frac{t-\delta_a}{\tau_a}}) & t \in [\delta_a, \delta_a + \delta_v] \\ \dfrac{I_a \tau_a}{C_{lv}}[(1 - \dfrac{\tau_a}{\tau_a - \tau_v}.e^{-\frac{\delta_v}{\tau_a}}).e^{-\frac{t-(\delta_a+\delta_v)}{\tau_v}} & \\ +\dfrac{\tau_v}{\tau_a - \tau_v}.e^{-\frac{t-\delta_a}{\tau_a}}] & t \in [\delta_a + \delta_v, +\infty[\end{array} \right. \quad (4)$$

The simplified circuit of a rc-circuit composed by n aggressors is shown in figure 9.

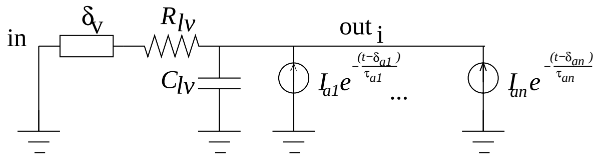

Fig. 9: Simplified circuit for a rc-circuit composed by n aggressors

3 Integration in a crosstalk evaluation tool

Let's consider an output signal out_i coupled with n active aggressors. The simplified circuit is shown in figure 10. R_v and C_v represent the victim driver. The resistor and capacitor of the aggressor drivers are take into account in the current sources.

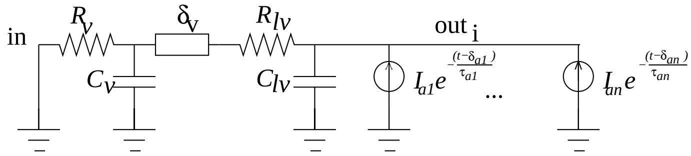

Fig. 10: Simplified circuit of an output signal out_i coupled with n active aggressors

We have seen in the introduction that the instability period can be computed with the gate delay and the instability periods of the input signals. But, the modelization of the rc-circuit adds some delay on the aggressor interconnection.

Thus, to determine which aggressors make a transition in the same time, we have to take into consideration the delay δ_a in the static instability analysis.
Finally, the simplified circuit for the crosstalk noise analysis is shown in figure 11.

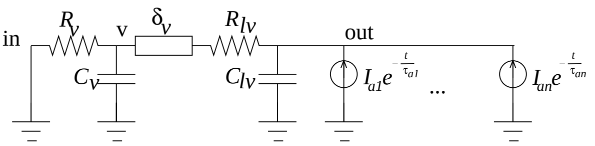

Fig. 11: Simplified circuit for crosstalk noise analysis

We have

$$\begin{cases} G_v.x_v + C_v.x'_v & = G_{lv}.x_{out} \\ G_{lv}.x_{out} + C_{lv}x'_{out} & = G_{lv}.x_v + \sum_{i=1}^{n} I_{ai}.e^{-\frac{t}{\tau_{ai}}} \end{cases} \tag{5}$$

where $G_v = \frac{1}{R_v} + \frac{1}{R_{lv}}$ et $G_{lv} = \frac{1}{R_{lv}}$
x_{out} can be written as

$$x_{out}(t) = K_1.e^{r_1.t} + K_2.e^{r_2.t} + \sum_{i=1}^{n} K_{ai}e^{-\frac{t}{\tau_{ai}}} \tag{6}$$

where τ_1, τ_2 are the time constants and depend of the values of the resistors and capacitors. K_1, K_2 and K_{ai} are some coefficient.
In order to determine the crosstalk noise of the signal out, we determine with the Newton-Raphson method the time t_{bruit} which $x'_{out}(t_{bruit}) = 0$. Then, the noise peak is

$$V_{bruit} = K_1.e^{r_1.t_{bruit}} + K_2.e^{r_2.t_{bruit}} + \sum_{i=1}^{n} K_{ai}e^{-\frac{t_{bruit}}{\tau_{ai}}} \tag{7}$$

4 Determining the parameters

4.1 Victim parameters

Let's consider the output signal of a rc-circuit composed by n nodes. When the input signal makes a transition from V_{SS} to V_{DD}, the output signal is

$$x_{out}(t) = \sum_{i=1}^{n} a_i e^{-h_i.t} + V_{DD} \tag{8}$$

The output signal obtained with the simplified circuit is :

$$\hat{x}_{out}(t) = \left| \begin{array}{ll} 0 & t \in [0, \delta_v] \\ V_{DD}.(1 - e^{-\frac{(t-\delta_v)}{\tau_v}}) & t \in [\delta_v, +\infty[\end{array} \right. \tag{9}$$

where $\tau_v = R_{lv}C_{lv}$
Figure 12 gives an example of the output signal obtained with the initial and simplified circuit.

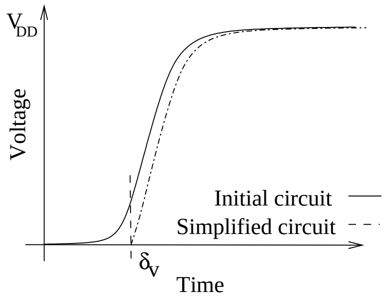

Fig. 12: Output signal obtained with the initial and simplified circuit

We have to determine τ_v and δ_v such as $\hat{x}_{out}$ is a good approximation of x_{out}. We note d the distance between two functions and defined as

$$d(f, g) = \int_0^{+\infty} (f(t) - g(t))^2.dt \tag{10}$$

g is a good approximation of f if the distance between f and g is minimal. We seek to define $\hat{x}_{out}$ such as $d(\hat{x}_{out}, x_{out})$ is minimal.

$$\begin{aligned} d(\hat{x}_{out}, x_{out}) &= \int_0^{+\infty} (x_{out}(t) - \hat{x}_{out}(t))^2 dt \\ &= \int_0^{\delta_v} (\sum_{i=1}^{n} a_i e^{-h_i.t} + V_{DD})^2 dt \\ &+ \int_{\delta_v}^{+\infty} (\sum_{i=1}^{n} a_i e^{-h_i.t} - V_{DD}.e^{-\frac{(t-\delta_v)}{\tau_v}})^2 dt \end{aligned} \tag{11}$$

In order to find the minimal value of $d(\hat{x}_{out}, x_{out})$ according to δ_v and τ_v we try to solve this equation system

$$\begin{cases} \frac{\partial}{\partial \delta_v} d(\hat{x}_{out}, x_{out}) & = & F_1(\delta_v, \tau_v) = 0 \\ \frac{\partial}{\partial \tau_v} d(\hat{x}_{out}, x_{out}) & = & F_2(\delta_v, \tau_v) = 0 \end{cases} \tag{12}$$

With

$$\begin{aligned} F_1(\delta_v, \tau_v) &= V_{DD}^2 - 2.V_{DD} \sum_{i=1}^{n} \frac{a_i}{1 + h_i\tau_v} e^{-h_i\delta_v} \\ F_2(\delta_v, \tau_v) &= \frac{V_{DD}^2}{2} - 2.V_{DD} \sum_{i=1}^{n} \frac{a_i}{(1 + h_i\tau_v)^2} e^{-h_i\delta_v} \end{aligned} \tag{13}$$

This equation system is not linear. So we use the Newton-Raphson method with partial derivation to solve this equation system. With an initial value of the solution $(\delta_{v_0}, \tau_{v_0})$ we determine $(\delta_{v_1}, \tau_{v_1}), \cdots, (\delta_{v_n}, \tau_{v_n})$ which converges to the solution. With an element $(\delta_{v_i}, \tau_{v_i})$, the next element is computed in solving the next matrix equation

$$\begin{pmatrix} \frac{\partial F_1}{\partial \delta_v}(\delta_{v_i}, \tau_{v_i}) & \frac{\partial F_1}{\partial \tau_v}(\delta_{v_i}, \tau_{v_i}) \\ \frac{\partial F_2}{\partial \delta_v}(\delta_{v_i}, \tau_{v_i}) & \frac{\partial F_2}{\partial \tau_v}(\delta_{v_i}, \tau_{v_i}) \end{pmatrix} \begin{pmatrix} \Delta\delta_v \\ \Delta\tau_v \end{pmatrix} = - \begin{pmatrix} F_1(\delta_{v_i}, \tau_{v_i}) \\ F_2(\delta_{v_i}, \tau_{v_i}) \end{pmatrix} \tag{14}$$

where

$$\begin{aligned}\Delta\delta_v &= \delta_{v_{i+1}} - \delta_{v_i} \\ \Delta\tau_v &= \tau_{v_{i+1}} - \tau_{v_i}\end{aligned}$$

4.2 Aggressor parameters

Let's consider the output signal obtained when the coupled aggressors are making a transition from V_{SS} to V_{DD}.

$$x_{out}(t) = \sum_{i=1}^{n} a_i e^{-h_i.t} \quad (15)$$

The output expression of the simplified circuit is

$$\hat{x}_{out}(t) = \left| \begin{array}{ll} 0 & t \in [0, \delta_a] \\ -\frac{I_a\tau_a}{C_{lv}}.(1 - e^{-\frac{t-\delta_a}{\tau_a}}) & t \in [\delta_a, \delta_a + \delta_v] \\ \frac{I_a\tau_a}{C_{lv}}[(1 - \frac{\tau_a}{\tau_a - \tau_v}.e^{-\frac{\delta_v}{\tau_a}}).e^{-\frac{t-(\delta_a+\delta_v)}{\tau_v}} & \\ +\frac{\tau_v}{\tau_a - \tau_v}.e^{-\frac{t-\delta_a}{\tau_a}}] & t \in [\delta_a + \delta_v, +\infty[\end{array} \right. \quad (16)$$

Figure 13 shows an example of output signal of initial and simplified circuit.

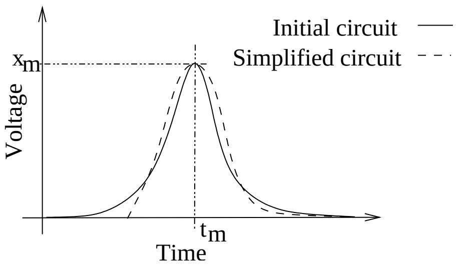

Fig. 13: Output signal of initial rc-circuit and of simplified circuit

Here, we used the same distance criteria to determine a good approxitaion of x_{out}.
For the case of a victim coupled with an aggressor the peak value is particularly important. Thus, we propose to minimize the distance between $\hat{x}_{out}$ and x_{out} with the next constraint :

- The peak noise obtained with the simplified circuit must occur at the same time and have the same value that the peak obtained with the initial circuit.

We obtain this equation system

$$\begin{cases} F_1(\delta_a, \tau_a, I_a) = \frac{\partial}{\partial \delta_a} d(\hat{x}_{out}, x_{out}) & = 0 \\ F_2(\delta_a, \tau_a, I_a) = x'_{out}(t_{max}) & = 0 \\ F_3(\delta_a, \tau_a, I_a) = \hat{x}_{out}(t_{max}) - x_{out}(t_{max}) & = 0 \end{cases} \quad (17)$$

The Newton-Raphson method using partial derivative solves this system.

4.3 Method to find the parameters with an electrical simulation

We have seen how to determine the parameters for the victim and for the aggressors with the output signal defined as the sum of exponential function. In a real size rc-circuit, the computation time necessary to find the exact formula of the output signal can be really important. Thus, we propose a technique to determine the parameters for both victim and aggressor with the result of an electrical simulation.
With an electrical simulation, we obtain a set of point that represent the output signal : x_{out} is defined for $i \in \{1, \cdots, n\}$ $x_{out}(t_i)$.
In order to approximate a function, we use the distance defined as

$$d(\hat{x}_{out}, x_{out}) = \sum_{i=1}^{N} (x_{out}(t_i) - \hat{x}_{out}(t_i))^2 \quad (18)$$

Then, we can compute the victim and aggressor parameters in replacing $\hat{x}_{out}$ with the output expression obtained with the simplified circuit and using the Newton-Raphson method.

5 Results

In this section we compare the results obtained with the initial rc-circuit and the simplified circuit.
Figure 14 shows the electrical simulation of the initial and simplified circuit. The initial circuit is composed by 100 nodes without aggressor. Its input makes a transition. We can see that the simplified circuit curve is near zero before the delay δ_v and after has a form of an exponential. The curves of the initial and simplified circuit are really closed.

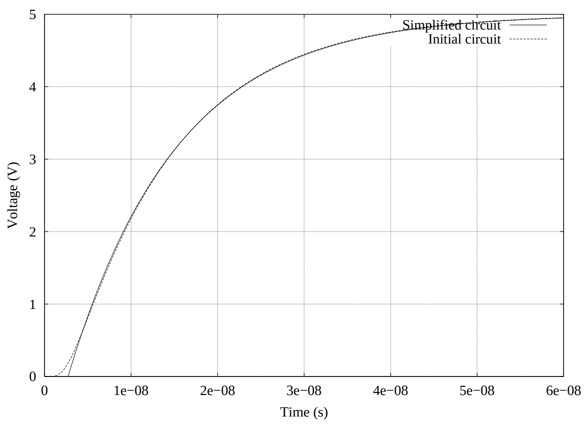

Fig. 14: Simplified circuit versus initial rc-circuit

Figure 15 shows the output signal of a rc-circuit composed by 100 nodes with one aggressor when the victim input signal is stable to V_{SS} and the aggressor input signal makes a transition from V_{SS} to V_{DD} and the simplified circuit.

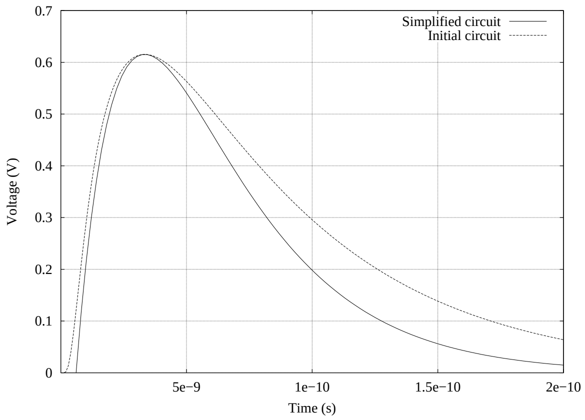

Fig. 15: Simplified circuit versus initial rc-circuit

We can see that the simplified circuit output is null for $t \in [0, \delta_a]$. Then, befor the peak, the curves obtained with the initial and simplified circuit are closed. The peak obtained with the two curves has the same value. But, after the peak, the simplified circuit gives a satisfying approximation of the initial circuit.
The table 1 presents the results obtained with the initial circuit (a victim coupled with two aggressors which are making a transition in the same time) and simplified circuit. The error is the difference of the peaks obtained with the simplified and initial circuit divided by V_{DD}.

$$Error = |\frac{Peak(simpl.\,circuit) - Peak(init.\,circuit)}{V_{DD}}| \tag{19}$$

	Peak value when V_{DD}=5V		
Number of node	Simplified circuit (V)	Initial circuit (V)	Error
50	1.01	0.91	1.98%
100	1.01	0.91	2.04%
150	0.78	0.91	2.68%
200	0.69	0.87	3.56%
250	0.59	0.72	2.70%
300	0.78	0.91	2.68%
350	0.49	0.91	8.45%

Table 1: Peak value and error obtained with the proposed method compared to an electrical simulation

We can see that the error is equal to 3.56% for a 200-nodes circuit and remains less than 10%.

6 Conclusion and future works

Signal integrity is becoming a major issue in the verification process of high performance designs. RC-circuit is one of the factors that may cause timing or functional failure in the circuit.
In this paper, we have presented a model to represent rc-circuit. This model is really simple and can easily be integrated in a crosstalk noise evaluation tool. The experience shows that the error of the proposed model remain less than 10%.
The method used to determine the simplified circuit parameters is based on the analytic output signal expression or on the results of an electrical simulation. In addition, we can apply this model on the results of methods, which determine the output waveform according of the input transition, such as the first three moments methods.
In the futur, some new parasitic elements will have a significant impact on verification tools : the inductance of interconnect. A more accurate model has to be develop taking into consideration the L of the interconnect and a method for computing its parameters.

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