Using Finite Impulse Response Feedback DACs to design ΣΔ modulators based on LC filters.

N. Beilleau, H. Aboushady and M.M. Louerat
University of Paris VI, LIP6/ASIM Laboratory
4 Place Jussieu, 75252 Paris, France
Email: Nicolas.Beilleau@lip6.fr, Hassan.Aboushady@lip6.fr

***Abstract*— This paper proposes a general method to design the noise-transfer-function of a continuous-time ΣΔ modulator having high-order passive LC loop filters. The feedback signals can have arbitrary shapes and can be applied either at internal nodes or only at the input node. As a design example, this technique is used to design a 4^{th} order ΣΔ modulator where the internal nodes of the loop filter are removed for a more efficient circuit implementation.**

I. Introduction

Nowadays, in Software Defined Radio (SDR) receivers, we try to displace the analog-to-digital conversion closer the antenna. The use of LC filters in a continuous-time (CT) bandpass (BP) ΣΔ modulator allows to consider the digitization of the signal directly at RF frequencies [1]. One problem, induced by using LC filters, is the loss of the parity between the order of the loop filter and the number of internal nodes (Fig.1). This implies limited degrees of freedom to design the noise-transfer-function (NTF) of the modulator. The different solutions proposed to solve this problem imply to add one additional feedback at each node of the loop filter of the modulator to compensate the lack in degrees of freedom [2][3]. These papers treated only LC-resonators based Sigma-Delta with rectangular pulse shape feedback digital-to-analog converters (DAC) applied to each node of the modulators.

In this paper we present a solution based on finite-impulse response feedback DAC (FIRDAC) to design CT modulators with n^{th} order LC filters (Fig.2). The FIRDACs have already been used to improve the stability of the high order ΣΔ modulators [4] or to decrease the effects of non-idealities like clock-jitter [5][6]. Here we use FIRDACs to provide the sufficient degrees of freedom for designing the NTF of the modulator.

In section II we will present the method to compute the FIRDAC coefficients, then we will show in section III that the shape of feedback signals can be either rectangular or non-rectangular. Finally as a design example, to illustrate this technique, we will show how to calculate the FIRDACs coefficients to obtain the required NTF of a ΣΔ modulator having a predefined 4^{th} order loop filter. The resulting ΣΔ is simulated and its performances are compared with a classical discrete-time (DT) 4^{th} order loop filter.

II. CT ΣΔ Modulators with FIRDAC

The method used to design the NTF is based on the identification between the loop gain of a CT modulator and the

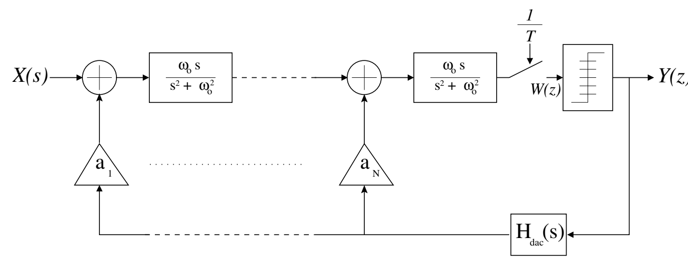

Fig. 1. N^{th} order continuous-time bandpass ΣΔ modulator based on LC filters using conventionnal feedback loops.

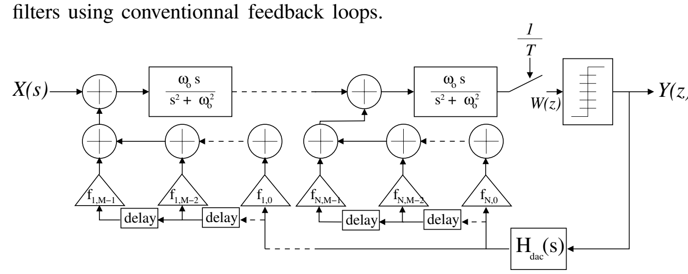

Fig. 2. N^{th} order continuous-time bandpass ΣΔ modulator based on LC filters using feedback loops with M^{th} order FIRDACs.

loop gain of its DT counterpart as previously done in [2][3][7]:

$$\begin{array}{rcl} G_d(z) & \equiv & G_c(z) = \frac{Y(z)}{W(z)} \\ & & G_c(z) = Z\{G_c(s)\} \end{array} \quad (1)$$

Our purpose is to determine the Z-transform of the CT modulator loop gain composed of FIRDACs and LC filters as shown Fig.2.

A FIRDAC is composed of a digital-to-analog converter (DAC) with gains f_j that are separated by half-cycle delays. The choice of using half-cycle delays instead of full-cycle delays as done in the classical architectures of FIR filter is to have more degrees of freedom. The transfer function of the FIRDAC can be expressed as:

$$H_{F_i}(s) = \underbrace{\sum_{j=0}^{M-1} f_{i,j}\, e^{-\frac{j}{2}sT}}_{F_i(s)} \; H_{DAC}(s) \quad (2)$$

where M is the order of the FIRDAC

We consider that the LC filters are ideal (i.e. infinite quality factor) and have the following transfer function:

$$H_{LC}(s) = \frac{\omega_o s}{s^2+\omega_o^2} \quad (3)$$

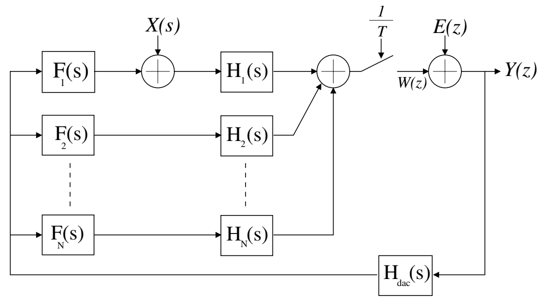

Fig. 3. Linear model of a CT ΣΔ modulator where the loop gain is considered as the addition of the feedback paths.

In order to simplify the calculations, we represent Fig.3 a general CT ΣΔ modulator where the loop gain is the addition of the feedback paths. This can be expressed as:

$$G_c(s) = \sum_{i=1}^{L} H_i(s)\, F_i(s)\, H_{dac}(s) \quad \text{with} \begin{cases} H_i(s) = \prod_{k=i}^{L-1} H_{LC}(s) \\ H_L(s) = 1 \end{cases} \tag{4}$$

Therefore the Z-transform of the loop gain has the following form:

$$Z\{G_c(s)\} = \sum_{i=1}^{L} \left(\sum_{j=0}^{M-1} \left[\underbrace{Z\left\{ f_{i,j}\, e^{-\frac{j}{2}sT} H_i(s)\, H_{dac}(s) \right\}}_{G_{i,j}(z)} \right] \right) \tag{5}$$

To minimize the number of z transform operations we try to get an expression that is not dependent of the FIRDAC order. Hence we express eq.(5) in the form:

$$Z\{G_c(s)\} = \sum_{i=1}^{L} \left(\sum_{j=0}^{\frac{M-1}{2}} \left[G_{i,2j}(z) + G_{i,2j+1}(z) \right] \right)$$
$$\text{with} \begin{cases} G_{i,2j}(z) = f_{i,2j}\, z^{-j} Z\left\{ H_i(s)\, H_{dac}(s) \right\} \\ G_{i,2j+1}(z) = f_{i,2j+1}\, z^{-j} Z\left\{ e^{-\frac{1}{2}sT} H_i(s)\, H_{dac}(s) \right\} \end{cases} \tag{6}$$

The conventional Z-transform cannot be used to represent any variations occuring between two consecutive sampling instants. Therefore we have to use the modified Z-transform method to express the half-period delay, the excess loop delay due to the circuit non-idealities or a RZ DAC signal [7] [8]. Equation (6) can be written as:

$$\begin{cases} G_{i,2j}(z) = f_{i,2j}\, z^{-j} Z_{m_1}\left\{ H_i(s)\, H_{dac}(s) \right\} \\ G_{i,2j+1}(z) = f_{i,2j+1}\, z^{-j} Z_{m_2}\left\{ H_i(s)\, H_{dac}(s) \right\} \end{cases}$$
$$\text{with} \quad m_1 = 1 - t_{delay} \text{ and } m_2 = 1 - t_{delay} - \frac{T}{2} \tag{7}$$

where t_{delay} includes the excess loop delay and any other delays in the shape of the feedback signal. In the next section we will consider different kinds of feedback signals and their influence on the determination of the FIRDAC coefficients.

III. Shape of the Feedback Signal

The feedback signals are usually rectangular (Fig.4) which are quite easy to design but are limited to the intermediate frequencies as it is illustrated in [9]. At high frequencies there are some advantages to work with sinusoidal signals (Fig.5), in

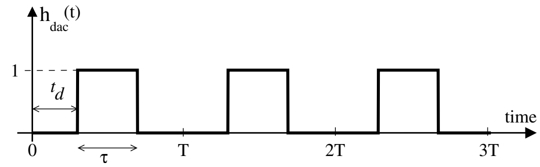

Fig. 4. Continuous-time rectangular feedback signal with T the sampling period.

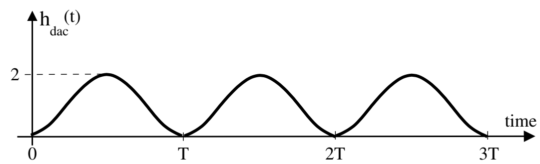

Fig. 5. Sine-shaped feedback signal ($h_{dac}(t) = 1 - \cos(w_{dac}t)$) with its period equal to the sampling period T.

[10] a sine-shaped feedback signal is presented as less sensitive to the clock jitter of the DAC. We will see in this section how to find the coefficients of a CT BP ΣΔ modulator with LC filters and rectangular or sine-shaped FIRDAC.

A. Rectangular Feedback Signals

The signal waveform depicted Fig.4 is expressed mathematically as:

$$h_{dac}(t) = u(t - t_d) - u(t - t_d - \tau) \tag{8}$$

where u(t) is the unit step function. We derive from eq.(8) the following transfer function by using the Laplace transform:

$$H_{dac}(s) = \frac{e^{-t_d s} - e^{-(t_d+\tau)s}}{s} \tag{9}$$

By introducing eq.(9) in eq.(7) we get:

$$\begin{cases} G_{i,2j}(z) = f_{i,2j}\, z^{-j} \left[Z_{m_1}\left\{ \frac{H_i(s)}{s} \right\} - Z_{m_2}\left\{ \frac{H_i(s)}{s} \right\} \right] \\ G_{i,2j+1}(z) = f_{i,2j+1}\, z^{-j} \left[Z_{m_3}\left\{ \frac{H_i(s)}{s} \right\} - Z_{m_4}\left\{ \frac{H_i(s)}{s} \right\} \right] \end{cases} \tag{10}$$

Neglecting the excess loop delay we have the following m_i:

$$\begin{cases} m_1 = 1 - \frac{t_d}{T} \\ m_2 = 1 - \frac{t_d}{T} - \frac{\tau}{T} \end{cases} \quad \begin{cases} m_3 = 1 - \frac{1}{2} - \frac{t_d}{T} \\ m_4 = 1 - \frac{1}{2} - \frac{t_d}{T} - \frac{\tau}{T} \end{cases} \tag{11}$$

The case of m_1 and m_2 has been already seen in [7]. The case of m_3 and m_4 is different because the delay of $\frac{T}{2}$ implies : $t_d + \tau \leq T/2$ to satisfy the modified Z-transform requirements. This leads to distinguish two cases:

- $t_d + \tau \leq T/2$: the loop gain is described by the equations (10) and (11).
- $t_d + \tau > T/2$: $G_{i,2j}(z)$ is still defined by eq. (10).

$$G_{i,2j+1}(z) = f_{i,2j+1}\, z^{-(j+1)} \left[Z_{m_3}\left\{ \frac{H_i(s)}{s} \right\} - Z_{m_4}\left\{ \frac{H_i(s)}{s} \right\} \right]$$
$$\text{with} \begin{cases} m_3 = 1 + \frac{1}{2} - \frac{t_d}{T} \\ m_4 = 1 + \frac{1}{2} - \frac{t_d}{T} - \frac{\tau}{T} \end{cases} \tag{12}$$

excess loop delay	$f_{1,0}$	$f_{1,1}$
0	$-\frac{1+\sqrt{2}}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
T	$-\frac{1}{\sqrt{2}}$	$\frac{1}{2-\sqrt{2}}$

TABLE I

FIRDAC COEFFICIENTS FOR A 2^{nd} ORDER CT BP $\Sigma\Delta$ MODULATOR BASED ON A LC FILTER USING A RZ FEEDBACK ($t_d = 0$ AND $\tau = \frac{T}{2}$).

excess loop delay	$f_{1,0}$	$f_{1,1}$	$f_{2,0}$	$f_{2,1}$
0	$-\frac{15}{16}$	$\frac{15}{16\sqrt{2}}$	0	$\frac{1}{4}$
T	0	$\frac{15}{16\sqrt{2}}$	0	$\frac{1}{4}$

TABLE II

FIRDAC COEFFICIENTS FOR A 2^{nd} ORDER CT BP $\Sigma\Delta$ MODULATOR BASED ON A LC FILTER USING A SINE-SHAPED FEEDBACK.

Hence we have determined the loop gain but not yet the coefficients. Starting from these equations we can find the coefficients of the modulator. We can apply this technique to a 2^{nd} order BP CT $\Sigma\Delta$ modulator with a RZ feedback signal. The delay of the feedback signal is $t_d = 0$ and the pulse width is $\tau = \frac{T}{2}$. Thus we are in the case of $t_d + \tau \leq T/2$ to determine the modified Z-transform:

$$\begin{cases} Z_{m_1}\left\{\frac{H_1(s)}{s}\right\} - Z_{m_2}\left\{\frac{H_1(s)}{s}\right\} = \frac{(1-\frac{\sqrt{2}}{2})z^{-1} - \frac{\sqrt{2}}{2}z^{-2}}{1+z^{-2}} \\ Z_{m_3}\left\{\frac{H_1(s)}{s}\right\} - Z_{m_4}\left\{\frac{H_1(s)}{s}\right\} = \frac{\frac{\sqrt{2}}{2}z^{-1} + (\frac{\sqrt{2}}{2}-1)z^{-2}}{1+z^{-2}} \end{cases} \quad (13)$$

with m_i determined by (11). The order of the FIRDAC is determined by the order of the modulator and the delay in the loop. As the delay of the FIRDAC is equal to 1 sampling-period T the number of coefficients has to be equal to the order of the modulator. In this case the order of the FIRDAC is 2, thus we derive from the equations (5), (10) and (13) the following expression for the CT modulator loop gain:

$$G_c(z) = \frac{\left((1-\frac{\sqrt{2}}{2})f_{1,0} + \frac{\sqrt{2}}{2}f_{1,1}\right)z^{-1} + \left(-\frac{\sqrt{2}}{2}f_{1,0} + (\frac{\sqrt{2}}{2}-1)f_{1,1}\right)z^{-2}}{1+z^{-2}} \quad (14)$$

We have now to identify the loop gain of the CT modulator with the one of the DT modulator:

$$G_d(z) = \frac{z^{-2}}{1+z^{-2}} \quad (15)$$

The denominators being the same we have only to compare the numerators of equ.(14) and equ.(15). Furthermore to make the computation of the coefficients automatic we use matrices:

$$\underbrace{\begin{pmatrix} 1-\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}-1 \end{pmatrix}}_{C} \underbrace{\begin{pmatrix} f_{1,0} \\ f_{1,1} \end{pmatrix}}_{F} = \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{D} \quad \begin{matrix} z^{-1} \\ z^{-2} \end{matrix}$$

It appears that if we multiply the CT loop gain by z^{-1} the equation is still solvable. This indicates that we can find coefficients for the same architecture of modulator but with an excess loop delay of 1 sampling-period T [8]. Therefore the previous matrices equation becomes:

$$\underbrace{\begin{pmatrix} 1-\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}-1 \end{pmatrix}}_{C} \underbrace{\begin{pmatrix} f_{1,0} \\ f_{1,1} \end{pmatrix}}_{F} = \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{D} \quad \begin{matrix} z^{-2} \\ z^{-3} \end{matrix}$$

The coefficients are found by solving the following equation and are listed in the table I:

$$F = C^{-1}\ D \quad (16)$$

B. Sine-Shaped Feedback Signal

The expressions of a sine-shaped signal are the following:

$$h_{dac}(t) = 1 - \cos(w_{dac}t) \Rightarrow H_{dac}(s) = \frac{w_{dac}^2}{s^2 + w_{dac}^2} \quad (17)$$

where: $w_{dac} = \frac{2\pi\, n_{cycles}}{T}$ and n_{cycles} the number of sinusoidal periods per sampling period.
We will show that the method used for a rectangular signal can be used for a sine-shaped signal. We consider a 2^{nd} order BP CT $\Sigma\Delta$ modulator with a sine-shaped feedback signal. Thus we have to determine the modified Z-transforms of the equation (7):

$$\begin{cases} Z_{m_1}\{H_1(s).H_{dac}(s)\} = \frac{16}{15}\ \frac{z^{-1}-z^{-2}}{1+z^{-2}} \\ Z_{m_2}\{H_1(s).H_{dac}(s)\} = \frac{16\sqrt{2}}{30}\ \frac{z^{-1}-z^{-3}}{1+z^{-2}} \end{cases} \text{with} \begin{cases} m_1 = 1 \\ m_2 = 1-\frac{1}{2} \end{cases} \quad (18)$$

These equations show that the loop gain of the CT $\Sigma\Delta$ modulator with sine-shaped feedback have an higher order than its DT equivalent described by equ.(15). The solution is to add a new FIRDAC just before sampling. Modified z-transform of this additionnal FIRDAC is:

$$\begin{cases} Z_{m_1}\{H_{dac}(s)\} = 0 \\ Z_{m_2}\{H_{dac}(s)\} = z^{-1} \end{cases} \text{with} \begin{cases} m_1 = 1 \\ m_2 = 1-\frac{1}{2} \end{cases} \quad (19)$$

As previously shown we find the loop gain of the CT modulator:

$$G_c(z) = \frac{(\frac{16}{15}f_{1,0} + \frac{16\sqrt{2}}{30}f_{1,1} + 2\,f_{2,1})z^{-1} - \frac{16}{15}f_{1,0}z^{-2} + (-\frac{16\sqrt{2}}{30}f_{1,1} + 2\,f_{2,1})z^{-3}}{1+z^{-2}} \quad (20)$$

We derive from equ.(15) and equ.(20) the following relationship:

$$\begin{pmatrix} \frac{16}{15} & \frac{16\sqrt{2}}{30} & 2 \\ -\frac{16}{15} & 0 & 0 \\ 0 & -\frac{16\sqrt{2}}{30} & 2 \end{pmatrix} \begin{pmatrix} f_{1,0} \\ f_{1,1} \\ f_{2,1} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{matrix} z^{-1} \\ z^{-2} \\ z^{-3} \end{matrix}$$

As previously we can also compute the coefficients for an excess loop delay of 1 sampling period. The coefficients for different excess loop delay are summarized in the table II.

IV. Design Example

At high frequencies the loop-filter circuit can be a major constraint to achieve the desired performances. Hence we can use the presented method to alleviate the problems due to the circuit of the loop-filter by adapting the modulator to a

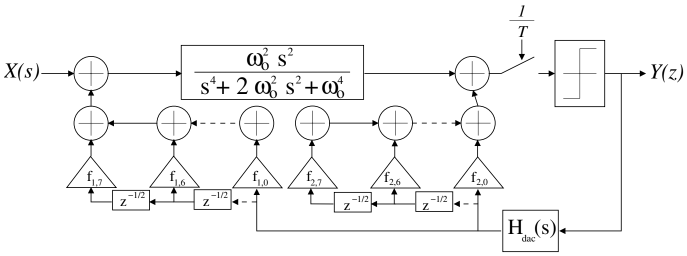

Fig. 6. 4^{th} order CT BP $\frac{f_s}{4}$ $\Sigma\Delta$ modulator based on a single block 4^{th} order filter and using FIRDACs.

$f_{1,0}$	$f_{1,1}$	$f_{1,2}$	$f_{1,3}$	$f_{1,4}$	$f_{1,5}$
0	0	2.7954	-3.0933	1.4975	5.2952
$f_{1,6}$	$f_{1,7}$	$f_{2,0}$	$f_{2,1}$	$f_{2,2}$	$f_{2,3}$
-6.1906	3.8977	0	0	1	-0.3406

TABLE III

COEFFICIENTS OF A CT BANDPASS $\Sigma\Delta$ MODULATOR WITH AN IN THE LOOP-FILTER AND SINUSOIDAL DACS.

predefined filter. We consider a $\Sigma\Delta$ modulator using a single-block 4^{th} order BP filter loop as depicted Fig.6. The loop filter has the following transfer function:

$$H_1(s) = \frac{\omega_o^2\, s^2}{s^4+2\,\omega_o^2\, s^2+\omega_o^4} \qquad (21)$$

Using this loop filter with conventional feedback loops (Fig.1) leads to lose 3 degrees of freedom to design the NTF comparing to a 4^{th} order BP filter based on integrators. The feedback signal is sine-shaped to reduce the sensitivity to clock jitter, thus the transfer function of the DAC is given by equ.(17). Therefore the use of FIRDACs allows to find the necessary degrees of freedom to design the NTF.

The order of the FIRDACs depends on the non-idealities taken into account as explained in the previous section. Hence to consider a loop-delay of 1 sampling period we need to have 8^{th} order FIRDACs. The modified Z-transforms of the loop-gain have the following forms:

$$\begin{cases} Z_{m_1}\{H_1(s).H_{dac}(s)\} = \\ \frac{0.90886z^{-4}-0.76664z^{-3}-0.76664z^{-2}+0.90886z^{-1}}{(1+z^{-2})^2} \\ Z_{m_2}\{H_1(s).H_{dac}(s)\} = \\ \frac{0.17479z^{-5}-0.69295z^{-4}-2.01994z^{-3}+0.69295z^{-2}+0.17479z^{-1}}{(1+z^{-2})^2} \end{cases}$$
$$\text{with} \begin{cases} m_1 = 1 \\ m_2 = 1-\frac{1}{2} \end{cases} \qquad (22)$$

The DT loop gain which is used to do the equivalence is the following:

$$G_d(z) = \frac{z^{-4}+2z^{-2}}{(1+z^{-2})^2} \qquad (23)$$

By using the presented method we deduce from equ.(22) the coefficients listed in the table III. We validate these coefficients by comparing the Signal-to-Noise Ratio (SNR) of this architecture and a 4^{th} order DT BP $\Sigma\Delta$ modulator. We can see from Fig.7 that the CT $\Sigma\Delta$ modulator determined with this method have the same performances as the DT modulator.

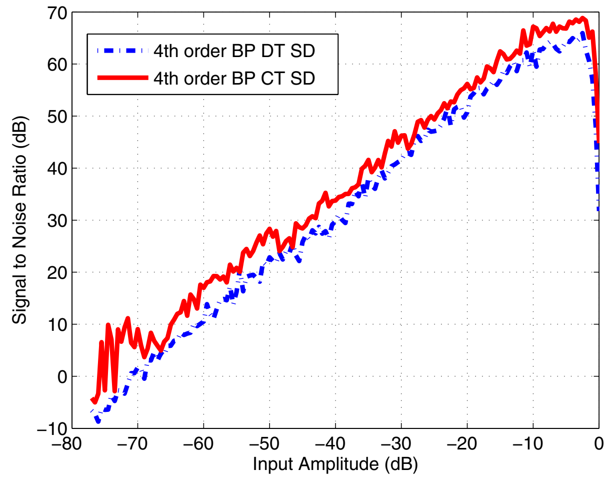

Fig. 7. SNR (dB) in function of the input's amplitude (dB) of a CT bandpass $\Sigma\Delta$ modulator and its DT counterpart. OSR = 58 and number of points N = 16384.

V. CONCLUSION

In this paper, we propose to add FIRDACs in the feedback loop of CT $\Sigma\Delta$ modulators with predefined loop filters having restricted degrees of freedom. This method is adapted to different feedback signal shapes as rectangular or not rectangular. A design example showed that, even with four times less degrees of freedom, we were able to design a modulator having the same NTF as an architecture based on integrators.

REFERENCES

[1] W. Gao and W. Snelgrove, ""A 950-MHz IF Second-Order Integrated LC Bandpass Delta-Sigma Modulator"," *IEEE J. Solid-State Circuits*, vol. 33, May 1998.

[2] O. Shoaei and W. M. Snelgrove, ""A multi-feedback design for LC bandpass Delta-Sigma modulators"," in *ISCAS'95*, May 1995.

[3] R. Maurino and P. Mole, ""A 200MHz IF 11-bit Fourth-Order Bandpass $\Sigma\Delta$ ADC in SiGe"," *IEEE J. Solid-State Circuits*, vol. 35, July 2000.

[4] Y. M. T. Okamoto and A. Yukawa, ""A Stable High-Order Delta-Sigma Modulator with a FIR Spectrum Distributor"," *IEEE J. Solid-State Circuits*, vol. 28, July 1993.

[5] O. Oliaei, ""Sigma-Delta Modulator with Spectrally Shaped Feedback"," *IEEE Trans. Circuit and Sys. II*, vol. 50, September 2003.

[6] B. Putter, ""$\Sigma\Delta$ ADC with Finite Impulse Response Feedback DAC"," in *ISSCC'04*, February 2004.

[7] H. Aboushady and M. Louërat, ""Systematic approach for discret-time to continuous-time tranformation of sigma-delta modulators"," in *ISCAS'02*, May 2002.

[8] H. Aboushady and M.-M. Louërat, ""Loop Delay Compensation in Bandpass Continuous-Time Sigma-Delta Modulators Without Additional Feedback Coefficients"," in *ISCAS'04*, May 2004.

[9] A. Latiri, H. Aboushady, and N.Beilleau, ""Design of Continuous-Time $\Sigma\Delta$ Modulators with Sine-Shaped Feedback DACs"," in *ISCAS'05*, May 2005.

[10] S. Luschas and H. Lee, ""High-Speed $\Sigma\Delta$ Modulators With Reduced Timing Jitter Sensitivity"," *IEEE Trans. Circuit and Sys. II*, vol. CAS-49, pp. 712–720, Nov. 2002.