Undersampled LC Bandpass ΣΔ Modulators with Feedback FIRDACs

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***Abstract*— A general technique for the design of undersampled LC bandpass modulators using feedback FIRDACs is proposed. The coefficients of the FIRDACs are used to increase the degrees of freedom in order to perform an exact equivalence between undersampled LC bandpass Sigma-Delta and high order Discrete-Time Sigma-Delta modulators. Using FIRDACs coefficients, it is also possible to simplify the circuit implementation by removing internal summing nodes and by decreasing coefficients spread. An undersampled 4th order LC Sigma-Delta is given as a design example. The effect of the undersampling ratio on the performance of finite quality factor LC Sigma-Delta modulators is also studied.**

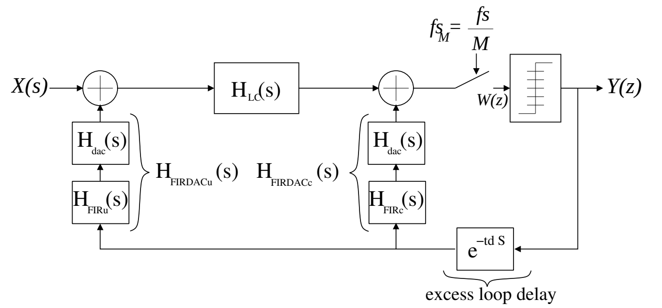

Fig. 1. LC bandpass ΣΔ modulator with an undersampling factor M.

I. Introduction

Recent years have shown an increasing interest to digitize the input signal near the front end of RF receivers so as to push more signal processing functions into the digital domain. LC filter based ΣΔ modulators have been considered for direct digitization at RF frequencies [1]–[3].

In order to simplify the digital downconversion, the sampling frequency, f_S, of a bandpass ΣΔ modulator is usually equal to four times the center frequency, f_o, [4]. At RF frequencies, this would significantly increase the sampling frequency, $f_S = 4f_o$, and would consequently increase the complexity and power consumption of the ΣΔ modulator and the subsequent digital circuits. Clock jitter is also a serious source of SNR degradation at high sampling frequencies [5].

Undersampling can be used to reduce the sampling frequency of Continuous-Time (CT) bandpass ΣΔ modulators, [6]–[8]. The design of CT ΣΔ modulators is usually based on Discrete-Time to Continuous-Time equivalence [9], [10]. Performing the equivalence between an integrator based Discrete-Time bandpass ΣΔ and an LC based CT ΣΔ modulator is a difficult task [9], [11]. This is mainly due to the fact that, in LC ΣΔ modulators, we have less coefficients than in the integrator based bandpass ΣΔ modulators.

In [12] and [13], a method to perform this equivalence using Finite Impulse Response DACs in the feedback loop, Fig.1, has been presented. In this paper, we propose to apply this method to undersampled LC ΣΔ modulators. It is shown that FIRDACs not only permit an exact equivalence between the Noise Transfer Function (NTF) of an undersampled LC ΣΔ modulator with the NTF of a conventional DT bandpass ΣΔ modulator, but it is also possible to increase the number of FIRDAC coefficients in order to have a more efficient circuit implementation. For example, the feedback coefficient spread can be reduced and, as shown in Fig.1, internal feedback nodes can be removed.

The main disadvantage of undersampled LC bandpass ΣΔ modulators is that they are more sensitive to the finite Q factor of the LC filter.

II. Undersampled LC ΣΔ modulators

We define the sampling frequency of an undersampled bandpass ΣΔ modulator as $f_{S_M} = \frac{f_S}{M}$, where f_S is the sampling frequency of a conventional bandpass ΣΔ modulator having a center frequency, $f_0 = \frac{f_S}{4}$, and M is the undersampling factor. The undersampled modulator quantization noise is shaped around a center frequency $f_{0_M} = \frac{f_0}{M}$. In Fig.2 and Fig.3, we plot the NTFs of LC bandpass ΣΔ modulator having odd and even undersampling factors, respectively. From these figures, we notice that the notch of the NTF is replicated at odd multiples of f_{0_M}, the maximum quantization noise power is replicated at even multiples of f_{0_M}. Since it is important that the ΣΔ modulator does not add noise in the vicinity of the input signal bandwidth centered around f_0, only odd values of the undersampling factor, M, can be used.

There are other restrictions on the value of the undersampling factor related to the feedback DAC signal. The transfer function of a conventional rectangular NRZ DAC signal is of the form, $\frac{sin(\pi f/f_{S_M})}{(\pi f/f_{S_M})}$, which has zeros at multiples of the sampling frequency f_{S_M}. This will cause a strong attenuation of the input signal located around $f_0 = \frac{M f_{S_M}}{4}$. To alleviate this problem, the ΣΔ feedback signal should be upconverted to the center frequency, f_0, of the input signal. This is done by mixing the ΣΔ output signal with a sinusoidal signal having a frequency, $f_{dac} = f_0 + \frac{f_{S_M}}{4}$, which leads to the following relation: $f_{dac} = (M+1)\frac{f_{S_M}}{4}$. If we define, N, as the ratio

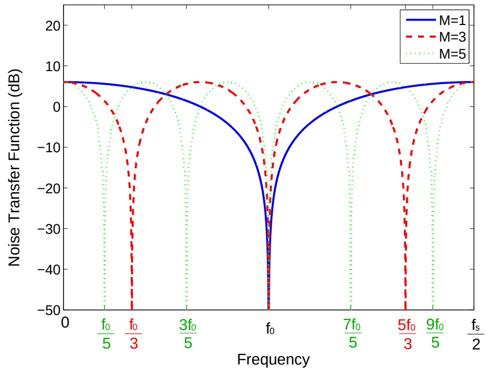

Fig. 2. NTF of LC $\Sigma\Delta$ modulators having odd undersampling factors. NTF notch is located at multiples of $\frac{f_0}{M}$.

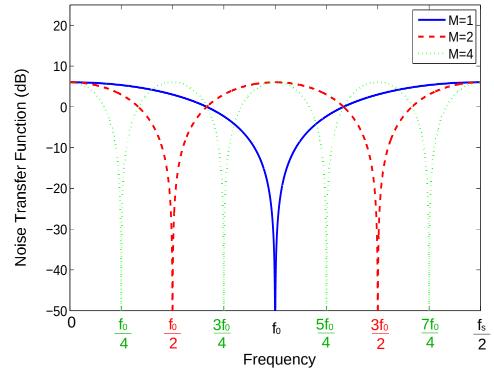

Fig. 3. NTF of LC $\Sigma\Delta$ modulators having even undersampling factors. Noise is added in the vicinity of the input signal bandwidth.

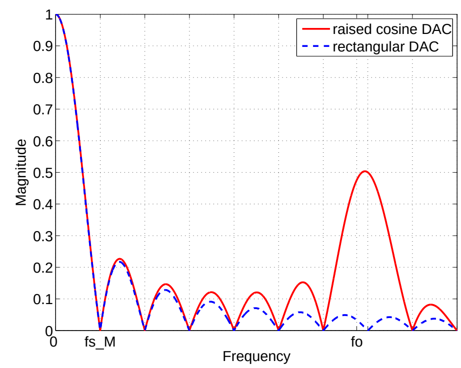

Fig. 4. Frequency responses of rectangular and raised cosine feedback DACs.

between the frequency of the DAC signal and the sampling frequency, we have: $N = \frac{f_{dac}}{f_{S_M}} = \frac{M+1}{4}$, which implies that: $M = 4N - 1$. In Fig.4, we can see the frequency responses of a rectangular DAC signal and a raised cosine DAC signal having $N = 7$.

III. Calculation of FIRDAC coefficients

The partial fraction expansion of a proper fraction[1] is unique. Therefore, the calculation of the FIRDACs coefficients being based on the DT-CT equivalence, we can identify the partial fractions derived from a CT loop gain z-transform, $G_c(z)$, with those derived from a DT loop gain counterpart, $G_d(z)$:

$$G_d(z) \equiv G_c(z) = \frac{Y(z)}{W(z)} \tag{1}$$

For LC based $\Sigma\Delta$ modulators, we place all the poles of the DT loop gain (i.e. the zeros of the NTF) at the same frequency without any optimization by spreading [4]. Hence, the partial fraction expansion of the DT loop gain is written as:

$$G_d(z) = \sum_{k=1}^{\frac{n}{2}} \left(\frac{\epsilon_k}{(z-\jmath)^k} + \frac{\epsilon_k^*}{(z+\jmath)^k} \right) \tag{2}$$

On the other hand, the CT loop gain derived from Fig.1 has the following form:

$$\begin{aligned} Gc(z) &= \underbrace{\mathcal{Z}\left\{H_{LC}(s)H_{DAC}(s)H_{FIR_U}(s)e^{-t_d s}\right\}}_{H_U(z)} \\ &- \underbrace{\mathcal{Z}\left\{H_{DAC}(s)H_{FIR_C}(s)e^{-t_d s}\right\}}_{H_C(z)} \end{aligned} \tag{3}$$

where t_d is the excess loop delay. $H_{LC}(s)$ is LC filter transfer function and is equal to $\left(\frac{\omega_0 s}{s^2+\omega_0^2}\right)^{\frac{n}{2}}$, where n is the order of the LC filter $(n = 2, 4, 6, ...)$ and ω_0 its center frequency. $H_{DAC}(s)$ is the DAC transfer function. In the case of a raised cosine: $H_{DAC}(s) = \frac{\omega_{dac}(1-e^{sT_M})}{s(s^2+\omega_{dac}{}^2)}$, with $T_M = \frac{1}{f_{S_M}}$. Note that the modified-z-transform technique was used to calculate the z-transforms in equation (3).

As depicted Fig.1, we consider only 2 feedback FIRDACs which have the following transfer functions:

$$\begin{cases} H_{FIR_U}(s) = \sum_{i=0}^{i=M_u-1} \quad u_i \quad e^{-\frac{isT_M}{2}} \\ H_{FIR_C}(s) = \sum_{i=0}^{i=M_c-1} \quad c_i \quad e^{-\frac{isT_M}{2}} \end{cases} \tag{4}$$

where M_u and M_c are respectively the order of the useful FIRDAC and the order of the compensation FIRDAC, and u_i and c_i their respective coefficients.

In equation (3), we define $H_U(z)$ and $H_C(z)$ as the useful transfer function and the compensation transfer function respectively. We can determine the partial fraction expansion of $H_U(z)$:

$$\begin{aligned} H_U(z) &= \underbrace{\sum_{i=1}^{D} \left(\frac{\gamma_{dac_i} + \gamma_{dac_i}^* + \gamma_{e_i}}{z^i} \right)}_{undesired\ term} \\ &+ \underbrace{\sum_{k=1}^{\frac{n}{2}} \left(\frac{\gamma_k}{(z - e^{r_1 T_M})^k} + \frac{\gamma_k^*}{(z + e^{r_2 T_M})^k} \right)}_{DT-CT\ equivalence} \end{aligned} \tag{5}$$

where r_1 and r_2 are the poles of $H_{LC}(s)$ and,

$$D = \lfloor \frac{M_u - 1}{2} + \frac{t_d}{T_M} \rfloor + 1$$

[1] a proper fraction is a fraction which has the order of its denominator higher than the order of its numerator.

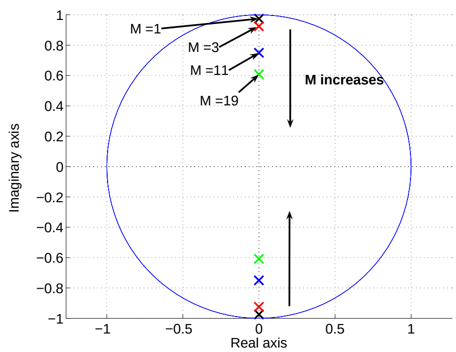

Fig. 5. Pole location of the LC $\Sigma\Delta$ loop gain, $G_c(z)$, for different undersampling factors, M, and a Q factor of 30.

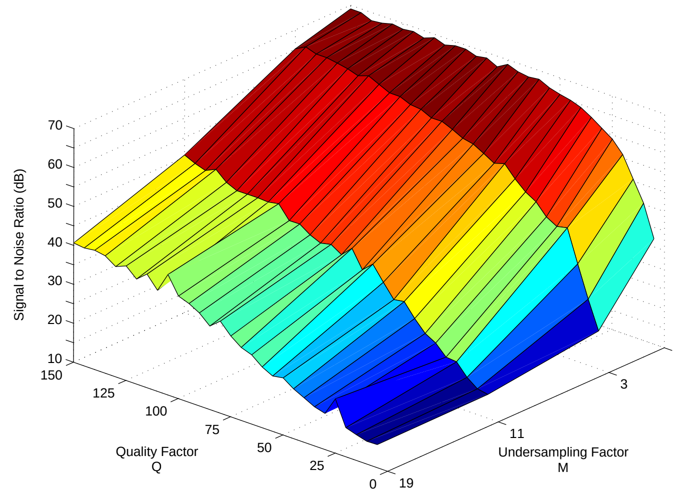

Fig. 6. Simulation results of maximum SNR of LC $\Sigma\Delta$ modulators having different undersampling factors and finite Q factors.

$\lfloor x \rfloor$ denotes the greatest integer less than or equal to x. Note that each γ_k coefficient is function of all the coefficients H_{FIR_U} and the excess loop delay t_d.
We use the second term of equation (5) to perform the DT-CT equivalence and to compute n coefficients H_{FIR_U}. The first term of equation (5) is undesired and has to be cancelled by the compensation transfer function which has the following partial fraction expansion:

$$H_C(z) = \sum_{i=0}^{i=M_c-1} \frac{c_i(1-cos(m_i 2\pi))}{z^{\ell_i+1}} \quad (6)$$

where $\ell_i = \lfloor \frac{i}{2} + \frac{t_d}{T_M} \rfloor$ and $m_i = 1 + \ell_i - (\frac{i}{2} + \frac{t_d}{T_M})$. We notice that when the excess loop delay is greater than 1 sampling period, the compensation transfer function, equation (6), does not cancel all the undesired terms of the equation (5). Therefore, we have to increase the order of the useful FIRDAC to cancel these terms by using its coefficients u_i:

$$M_u = n + \lfloor \frac{t_d}{T_M} \rfloor$$

Hence the number of coefficients c_i depends also on the excess loop delay [13]:

$$M_c = 2\left(D - \lfloor \frac{t_d}{T_M} \rfloor\right)$$

It is also possible to increase the order of the useful FIRDAC to offer additionnal degrees of freedom which can be used to relax some circuit level specifications. This will be illustrated by a design example in section V.

IV. LC $\Sigma\Delta$ Modulators with Finite Q Factor

All the previous analysis for the determination of CT coefficients have been led for an infinite Q factor. However, in practice the value of the quality factor is finite, leading to:

$$H_{LC}(s) = \left(\frac{\omega_0 s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}\right)^{\frac{n}{2}} \quad (7)$$

From equation (7), we find the poles of $H_{LC}(s)$:

$$r_{1,2} = \frac{1}{2}\left(-\frac{\omega_0}{Q} \pm j\sqrt{-\frac{\omega_0^2}{Q^2} + 4\omega_0^2}\right) \quad (8)$$

For a 4^{th} order $\Sigma\Delta$ modulator, the useful transfer function can be written as:

$$\begin{aligned} H_U(z) &= \frac{\gamma_1}{z - e^{r_1 T_M}} + \frac{\gamma_1^*}{z - e^{r_2 T_M}} \\ &+ \frac{\gamma_2}{(z - e^{r_1 T_M})^2} + \frac{\gamma_2^*}{(z - e^{r_2 T_M})^2} + \sum_{i=1}^{i=D} \frac{\gamma_{e_i}}{z^i} \end{aligned} \quad (9)$$

Comparing $H_U(z)$ in equation (9) to $G_d(z)$, the DT $\Sigma\Delta$ loop gain, defined by equation (2), we notice that it is impossible to identify ϵ_1 to γ_1 because $e^{r_1 T_M}$ cannot be equated to j for a finite quality factor. In fact,

$$|e^{r_1 T_M}| = e^{-\frac{\omega_0 T_M}{2Q}} \neq 1 = |j| \quad (10)$$

It is then impossible to achieve DT-CT equivalence in the case of finite Q factor LC filters with the model described in Fig.1.

In the following, we shall analyse the influence of finite Q factor on the performance of LC $\Sigma\Delta$ modulators where the FIRDAC coefficients have been calculated assuming ideal LC filters having infinite Q factor.

Using equation (8), we can find expressions for the poles of, $G_c(z)$, the CT loop gain:

$$p_{1,2} = e^{\left(\frac{-\omega_0}{2Q} \pm \frac{1}{2} \jmath \sqrt{-\frac{\omega_0^2}{Q^2} + 4\omega_0^2}\right) T_M} \quad (11)$$

Substituting $\omega_0 T_M$ by $\frac{2\pi f_0}{f_{S_M}} = \frac{M\pi}{2}$, the expression of $p_{1,2}$ becomes:

$$p_{1,2} = e^{-\frac{M\pi}{4}\left(\frac{1}{Q} \pm \jmath \sqrt{-\frac{1}{Q^2} + 4}\right)} \quad (12)$$

Since Q $\gg$ 1, the term $\sqrt{-\frac{1}{Q^2} + 4}$ is very close to 2. Therefore:

$$p_{1,2} \approx e^{-\frac{M\pi}{4Q}} e^{\pm \jmath \frac{M\pi}{2}} = \pm \jmath e^{-\frac{M\pi}{4Q}} \quad (13)$$

From equation (13), we can see that the poles of the loop gain of LC $\Sigma\Delta$ modulators having finite Q factor are no longer located at the unit circle as it has been the case for an infinite

TABLE I

FEEDBACK FIRDAC COEFFICIENTS FOR AN UNDERSAMPLED 4^{th} ORDER LC $\Sigma\Delta$ MODULATOR, $M = 3$, $OSR = 58$, $t_d = \frac{3T_M}{2}$.

FIRDAC coefficients	value
u_0	$(-0.212\lambda + 0.281)$
u_1	$(-0.6\lambda - 1.810)$
u_2	$(-0.849\lambda - 2.531)$
u_3	$(-0.6\lambda - 1.770)$
u_4	-0.212λ
c_0	$-0.023\lambda - 0.970$
c_2	$+0.0696\lambda + 0.179$
c_4	0.024λ

Q factor. This is illustrated in Fig.5, where we can see the influence of the undersampling ratio, M, on the position of the loop gain poles. As M increases, the poles get further away from the unit circle, which will result in a lower attenuation of the $\Sigma\Delta$ quantization noise.

Fig.6, shows simulation results of the maximum SNR of a 4^{th} order LC $\Sigma\Delta$ modulators having different Q factors and undersampling factors. It is obvious from this figure that the SNR degradation due to finite Q factor is more severe for higher undersampling factors. This can be explained intuitively, from Fig.2, where we can see that, as M increases, the notch the NTF becomes sharper around f_0 thus requiring a higher Q factor. Using equation (13), we can calculate that an LC $\Sigma\Delta$ having $M = 3$ will require a quality factor 3 times higher than an LC $\Sigma\Delta$ having $M = 1$. Equation (13), has been verified by the simulation results shown in Fig.6.

V. DESIGN EXAMPLE

In the following, we present an undersampled $\Sigma\Delta$ 4^{th} order LC $\Sigma\Delta$ modulator with parametric FIRDAC coefficients. The modulator has an undersampling ratio, $M = 3$, an oversampling ratio, $OSR = 58$ and an excess loop delay, $t_d = \frac{3T_M}{2}$. The design method described in section III has been used to calculate the coefficients of the useful and the compensation FIRDACs. These coefficients are listed in table I. Note that we have increased the order M_u of the useful FIRDAC, H_{FIR_U}, in order to have one additionnal degree of freedom, λ, which can be used to relax some circuit level specifications. For example, we can reduce the coefficients spread from 2.8 for $\lambda = 0$ to 1.1 when $\lambda = -3.0$. Fig.7, shows the SNR of the undersampled LC $\Sigma\Delta$ modulator with feedback FIRDACs having different values of λ. This is compared to the SNR of a 4^{th} DT bandpass $\Sigma\Delta$. The different modulators give very similar simulation results.

VI. CONCLUSION

In this paper, we presented a general method for the design of undersampled LC $\Sigma\Delta$ modulators. In order to perform NTF equivalence with conventional DT bandpass $\Sigma\Delta$ modulators, it is proposed to add FIRDACs in the feedback loop. Using this technique, internal summing nodes can been removed and coefficients spread can be reduced at the expense of higher FIRDACs order. The SNR degradation due to finite Quality factor of the LC filters is studied theoretically and confirmed by simulation results. In order to validate the proposed technique, an undersampled $4th$ order LC $\Sigma\Delta$ modulator, with M=3, having exactly the same NTF as a DT integrator based $\Sigma\Delta$ modulator is designed and simulated.

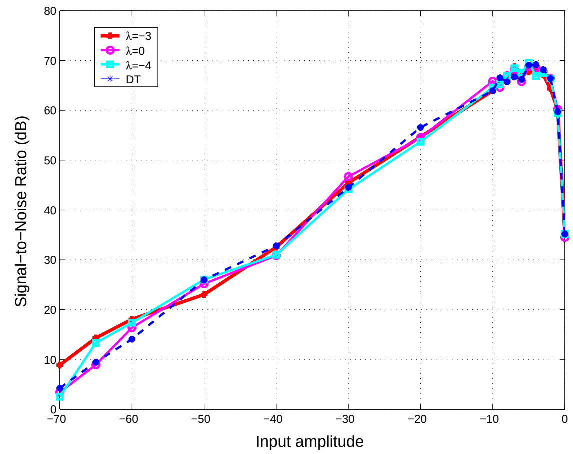

Fig. 7. Simulation results of 4^{th} order DT and undersampled LC $\Sigma\Delta$ modulators using different values for λ (table I).

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