TABLE 5.1   (Unilateral) z-Transform Pairs

No.	$x[n]$	$X[z]$
1	$\delta[n-n]$	z^{-k}
2	$u[n]$	$\dfrac{z}{z-1}$
3	$nu[n]$	$\dfrac{z}{(z-1)^2}$
4	$n^2 u[n]$	$\dfrac{z(z+1)}{(z-1)^3}$
5	$n^3 u[n]$	$\dfrac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^n u[n]$	$\dfrac{z}{z-\gamma}$
7	$\gamma^{n-1} u[n-1]$	$\dfrac{1}{z-\gamma}$
8	$n\gamma^n u[n]$	$\dfrac{\gamma z}{(z-\gamma)^2}$
9	$n^2 \gamma^n u[n]$	$\dfrac{\gamma z(z+\gamma)}{(z-\gamma)^3}$
10	$\dfrac{n(n-1)(n-2)\cdots(n-m+1)}{\gamma^m m!}\gamma^n u[n]$	$\dfrac{z}{(z-\gamma)^{m+1}}$
11a	$\|\gamma\|^n \cos \beta n\, u[n]$	$\dfrac{z(z-\|\gamma\|\cos\beta)}{z^2-(2\|\gamma\|\cos\beta)z+\|\gamma\|^2}$
11b	$\|\gamma\|^n \sin \beta n\, u[n]$	$\dfrac{z\|\gamma\|\sin\beta}{z^2-(2\|\gamma\|\cos\beta)z+\|\gamma\|^2}$
12a	$r\|\gamma\|^n \cos(\beta n+\theta)u[n]$	$\dfrac{rz[z\cos\theta-\|\gamma\|\cos(\beta-\theta)]}{z^2-(2\|\gamma\|\cos\beta)z+\|\gamma\|^2}$
12b	$r\|\gamma\|^n \cos(\beta n+\theta)u[n] \qquad \gamma=\|\gamma\|e^{j\beta}$	$\dfrac{(0.5re^{j\theta})z}{z-\gamma}+\dfrac{(0.5re^{-j\theta})z}{z-\gamma^*}$
12c	$r\|\gamma\|^n \cos(\beta n+\theta)u[n]$	$\dfrac{z(Az+B)}{z^2+2az+\|\gamma\|^2}$

$$r = \sqrt{\frac{A^2|\gamma|^2 + B^2 - 2AaB}{|\gamma|^2 - a^2}}$$

$$\beta = \cos^{-1}\frac{-a}{|\gamma|}$$

$$\theta = \tan^{-1}\frac{Aa-B}{A\sqrt{|\gamma|^2 - a^2}}$$

TABLE 5.2 z-Transform Operations

Operation	$x[n]$	$X[z]$
Addition	$x_1[n] + x_2[n]$	$X_1[z] + X_2[z]$
Scalar multiplication	$ax[n]$	$aX[z]$
Right-shifting	$x[n-m]u[n-m]$	$\dfrac{1}{z^m}X[z]$
	$x[n-m]u[n]$	$\dfrac{1}{z^m}X[z] + \dfrac{1}{z^m}\displaystyle\sum_{n=1}^{m}x[-n]z^n$
	$x[n-1]u[n]$	$\dfrac{1}{z}X[z] + x[-1]$
	$x[n-2]u[n]$	$\dfrac{1}{z^2}X[z] + \dfrac{1}{z}x[-1] + x[-2]$
	$x[n-3]u[n]$	$\dfrac{1}{z^3}X[z] + \dfrac{1}{z^2}x[-1] + \dfrac{1}{z}x[-2] + x[-3]$
Left-shifting	$x[n+m]u[n]$	$z^m X[z] - z^m \displaystyle\sum_{n=0}^{m-1}x[n]z^{-n}$
	$x[n+1]u[n]$	$zX[z] - zx[0]$
	$x[n+2]u[n]$	$z^2 X[z] - z^2 x[0] - zx[1]$
	$x[n+3]u[n]$	$z^3 X[z] - z^3 x[0] - z^2 x[1] - zx[2]$
Multiplication by γ^n	$\gamma^n x[n]u[n]$	$X\left[\dfrac{z}{\gamma}\right]$
Multiplication by n	$nx[n]u[n]$	$-z\dfrac{d}{dz}X[z]$
Time convolution	$x_1[n] * x_2[n]$	$X_1[z]X_2[z]$
Time reversal	$x[-n]$	$X[1/z]$
Initial value	$x[0]$	$\displaystyle\lim_{z\to\infty} X[z]$
Final value	$\displaystyle\lim_{N\to\infty} x[N]$	$\displaystyle\lim_{z\to1}(z-1)X[z]$ poles of $(z-1)X[z]$ inside the unit circle