

Lecture II

- **Introduction**
- **Negative Resistance Oscillators**
- **Integrated Passive Components**
- **Phase Noise in Local Oscillators**

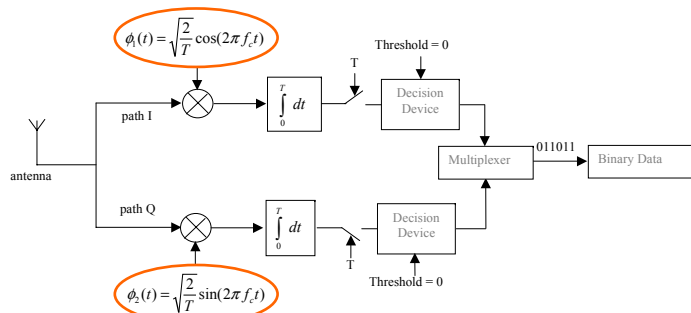
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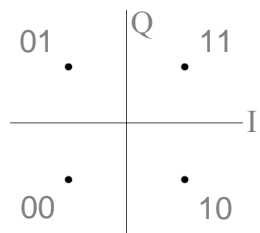
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QPSK Receiver



QPSK Constellation Diagram



References

- M. Perrott, "High Speed Communication Circuits and Systems", M.I.T. OpenCourseWare, <http://ocw.mit.edu/>, Massachusetts Institute of Technology, 2003.
- T. Lee, "The Design of CMOS Radio-Frequency Integrated Circuits", Cambridge University Press, 2004.
- B. Razavi, "Design of Analog CMOS Integrated Circuits", Mc Graw-Hill, 2001.

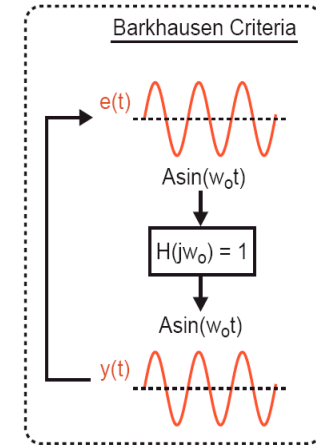
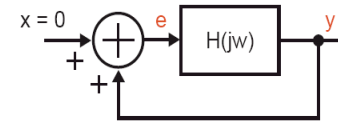
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Barkhausen's Criteria for Oscillation



- **Closed loop transfer function**

$$G(jw) = \frac{Y(jw)}{X(jw)} = \frac{H(jw)}{1 - H(jw)}$$

- **Self-sustaining oscillation at frequency w_0 if**

$$H(jw_0) = 1$$

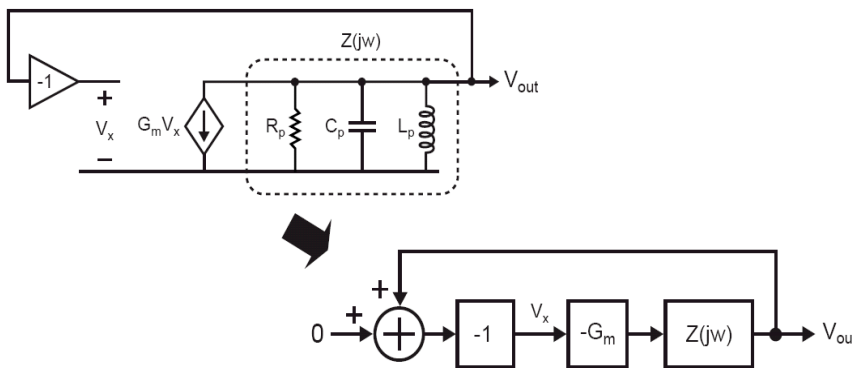
- **Amounts to two conditions:**

- Gain = 1 at frequency w_0
- Phase = $n360$ degrees ($n = 0, 1, 2, \dots$) at frequency w_0

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Resonator-Based Oscillator



- **Barkhausen Criteria for oscillation at frequency w_0 :**

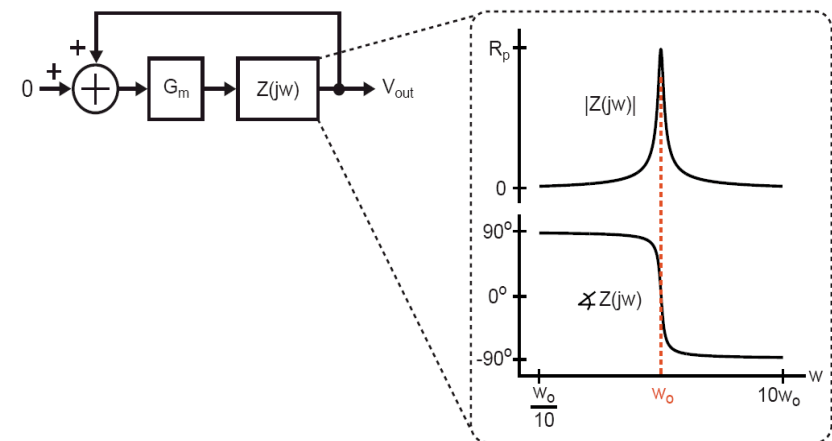
$$\overline{G_m Z(jw_0)} = 1$$

- Assuming G_m is purely real, $Z(jw_0)$ must also be purely real

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A Closer Look At Resonator-Based Oscillator



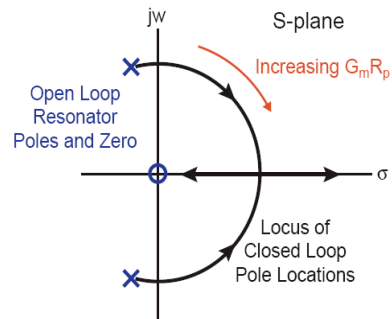
- **For parallel resonator at resonance**

- **Looks like resistor (i.e., purely real) at resonance**
 - Phase condition is satisfied
 - Magnitude condition achieved by setting $G_m R_p = 1$

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Impact of Different G_m Values

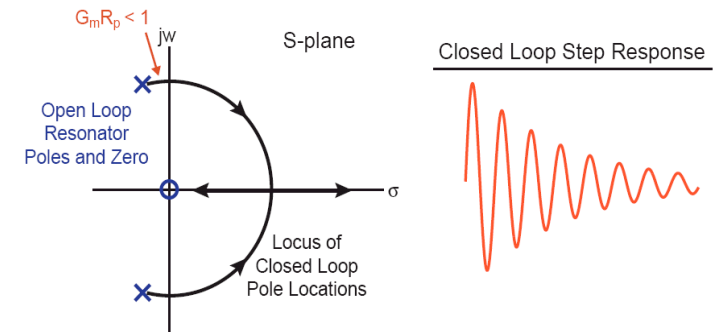


- Root locus plot allows us to view closed loop pole locations as a function of open loop poles/zero and open loop gain ($G_m R_p$)
 - As gain ($G_m R_p$) increases, closed loop poles move into right half S-plane

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Impact of Setting G_m too low

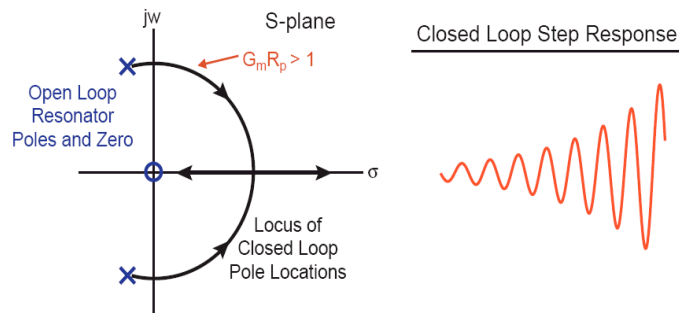


- Closed loop poles end up in the left half S-plane
 - Underdamped response occurs
 - Oscillation dies out

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Impact of Setting G_m too High

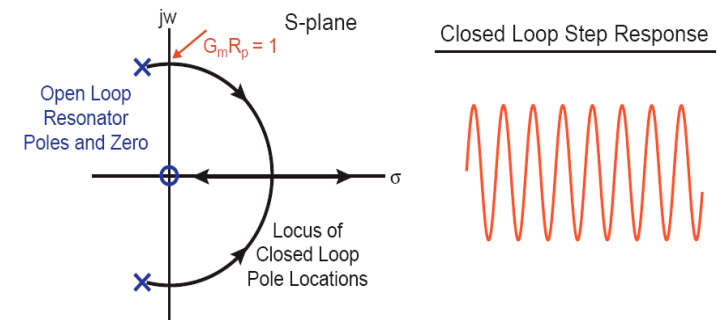


- Closed loop poles end up in the right half S-plane
 - Unstable response occurs
 - Waveform blows up!

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Setting G_m To Just the Right Value

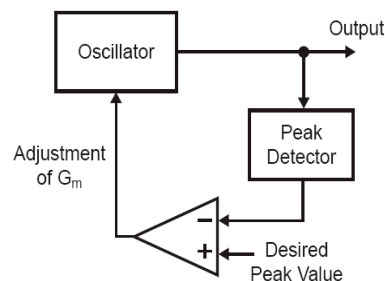


- Closed loop poles end up on $j\omega$ axis
 - Oscillation maintained
- Issue – $G_m R_p$ needs to *exactly* equal 1
 - How do we achieve this in practice?

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Amplitude Feedback Loop

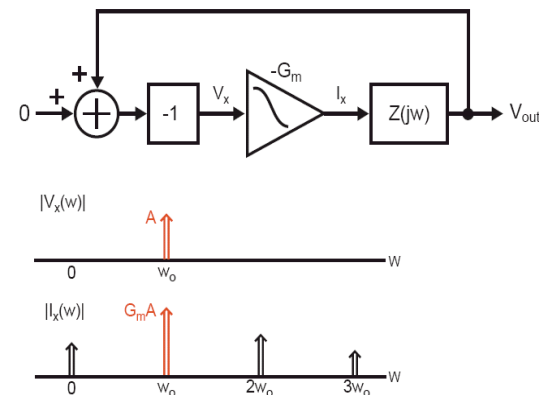


- One thought is to detect oscillator amplitude, and then adjust G_m so that it equals a desired value
 - By using feedback, we can precisely achieve $G_m R_p = 1$
- Issues
 - Complex, requires power, and adds noise

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Leveraging Amplifier Nonlinearity as Feedback

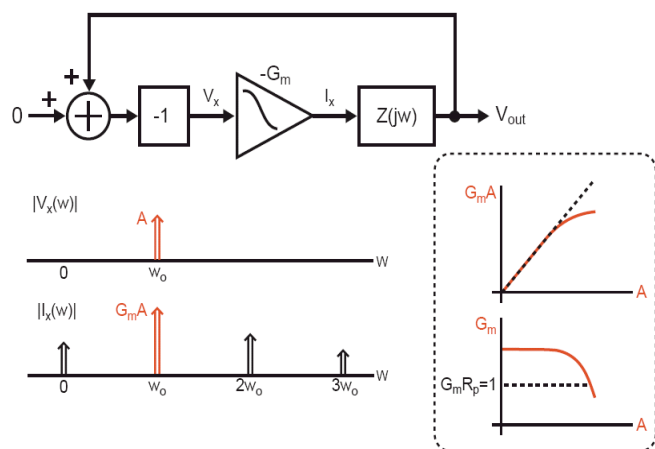


- Practical transconductance amplifiers have saturating characteristics
 - Harmonics created, but filtered out by resonator
 - Our interest is in the relationship between the input and the fundamental of the output

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Leveraging Amplifier Nonlinearity as Feedback

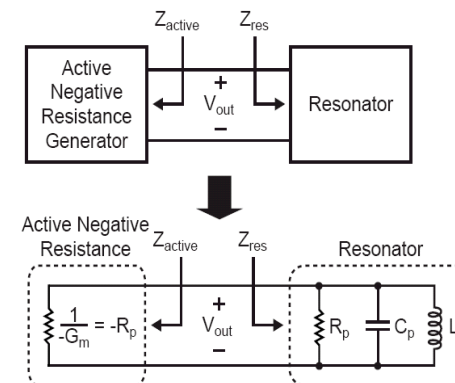


- As input amplitude is increased
 - Effective gain from input to fundamental of output drops
 - Amplitude feedback occurs! ($G_m R_p = 1$ in steady-state)

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One-Port View of Resonator-Based Oscillators



- Convenient for intuitive analysis
- Here we seek to cancel out loss in tank with a negative resistance element
 - To achieve sustained oscillation, we must have

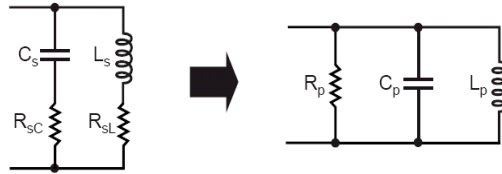
$$\frac{1}{G_m} = R_p \Rightarrow G_m R_p = 1$$

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One-Port Modeling Requires Parallel RLC Network

- Since VCO operates over a very narrow band of frequencies, we can always do series to parallel transformations to achieve a parallel network for analysis

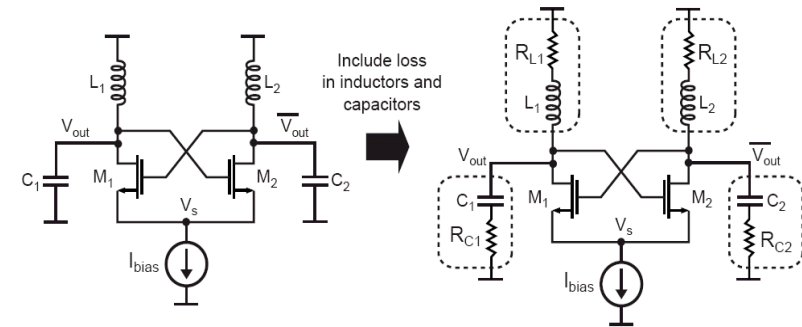


- Warning – in practice, RLC networks can have secondary (or more) resonant frequencies, which cause undesirable behavior
 - Equivalent parallel network masks this problem in hand analysis
 - Simulation will reveal the problem

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Example – Negative Resistance Oscillator

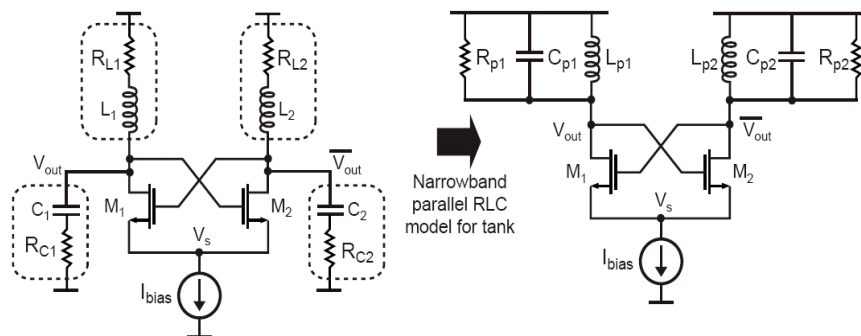


- This type of oscillator structure is quite popular in current CMOS implementations
 - Advantages
 - Simple topology
 - Differential implementation (good for feeding differential circuits)
 - Good phase noise performance can be achieved

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Analysis of Negative Resistance Oscillator (Step 1)

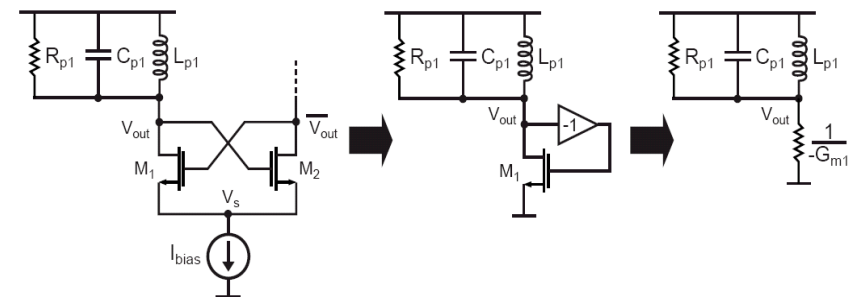


- Derive a parallel RLC network that includes the loss of the tank inductor and capacitor
 - Typically, such loss is dominated by series resistance in the inductor

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Analysis of Negative Resistance Oscillator (Step 2)

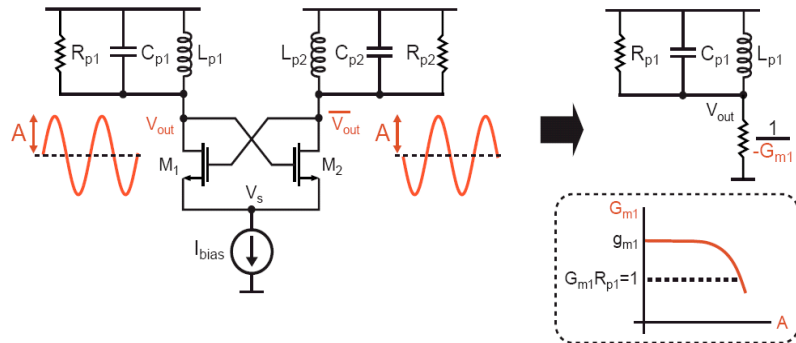


- Split oscillator circuit into half circuits to simplify analysis
 - Leverages the fact that we can approximate V_s as being incremental ground (this is not quite true, but close enough)
- Recognize that we have a diode connected device with a negative transconductance value
 - Replace with negative resistor
 - Note: G_m is *large signal* transconductance value

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Design of Negative Resistance Oscillator

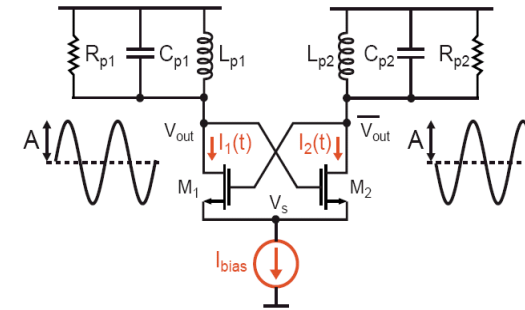


- Design tank components to achieve high Q
 - Resulting R_p value is as large as possible
- Choose bias current (I_{bias}) for large swing (without going far into saturation)
 - We'll estimate swing as a function of I_{bias} shortly
- Choose transistor size to achieve adequately large g_{m1}
 - Usually twice as large as $1/R_{p1}$ to guarantee startup

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Calculation of Oscillator Swing



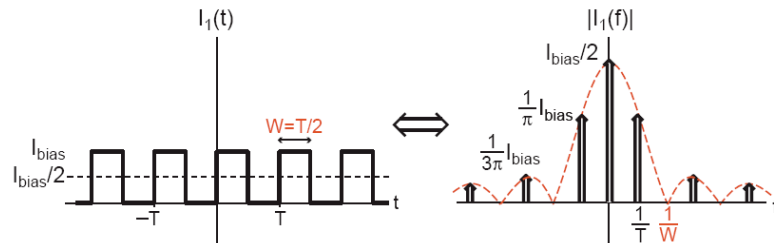
- Design tank components to achieve high Q
 - Resulting R_p value is as large as possible
- Choose bias current (I_{bias}) for large swing (without going far into saturation)
 - We'll estimate swing as a function of I_{bias} in next slide
- Choose transistor size to achieve adequately large g_{m1}
 - Usually twice as large as $1/R_{p1}$ to guarantee startup

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Calculation of Oscillator Swing as a Function of I_{bias}

- By symmetry, assume $I_1(t)$ is a square wave
 - We are interested in determining fundamental component
 - (DC and harmonics filtered by tank)



- Fundamental component is

$$I_1(t) \Big|_{\text{fundamental}} = \frac{2}{\pi} I_{bias} \sin(\omega_o t), \quad \text{where } \omega_o = \frac{2\pi}{T}$$

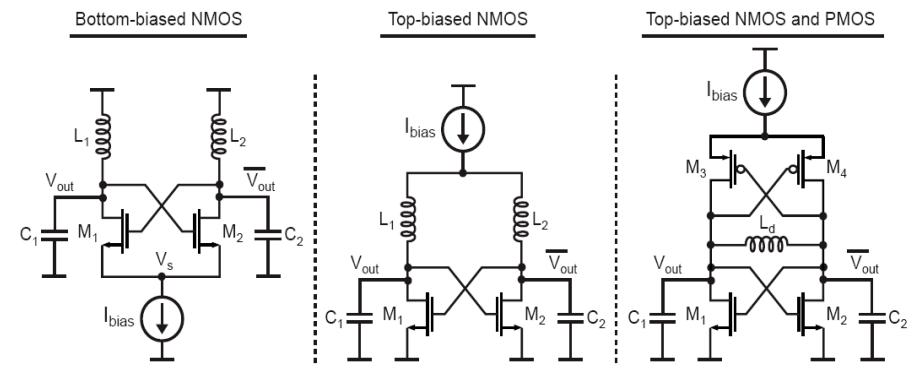
- Resulting oscillator amplitude

$$A = \frac{2}{\pi} I_{bias} R_p$$

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Variations on a Theme

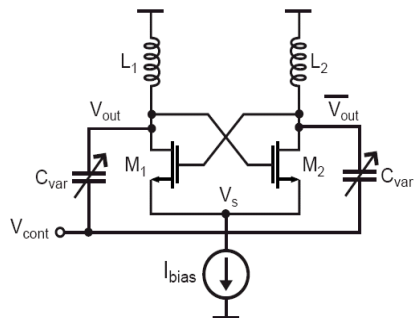


- Biasing can come from top or bottom
- Can use either NMOS, PMOS, or both for transconductor
 - Use of both NMOS and PMOS for coupled pair would appear to achieve better phase noise at a given power dissipation
 - See Hajimiri et. al, "Design Issues in CMOS Differential LC Oscillators", JSSC, May 1999 and Feb, 2000 (pp 286-287)

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Voltage Controlled Oscillators (VCO's)



- Include a tuning element to adjust oscillation frequency
 - Typically use a variable capacitor (varactor)
- Varactor incorporated by replacing fixed capacitance
 - Note that much fixed capacitance cannot be removed (transistor junctions, interconnect, etc.)
 - Fixed cap lowers frequency tuning range

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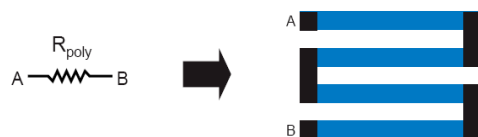
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Polysilicon Resistors

- Use unsilicided polysilicon to create resistor



- Key parameters
 - Resistance (usually 100- 200 Ohms per square)
 - Parasitic capacitance (usually small)
 - Appropriate for high speed amplifiers
 - Linearity (quite linear compared to other options)
 - Accuracy (usually can be set within $\pm 15\%$)

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MOS Resistors

- Bias a MOS device in its triode region



$$R_{ds} \approx \frac{1}{\mu C_{ox} W/L ((V_{gs} - V_T) - V_{DS})}$$

- High resistance values can be achieved in a small area (MegaOhms within tens of square microns)
- Resistance is quite nonlinear
 - Appropriate for small swing circuits

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High Density Capacitors (Biasing, Decoupling)

- MOS devices offer the highest capacitance per unit area
 - Limited to a one terminal device
 - Voltage must be high enough to invert the channel



Key parameters

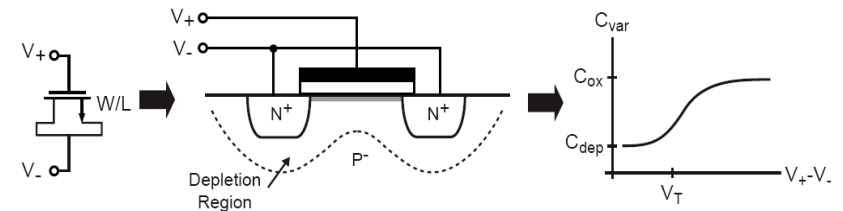
- Capacitance value
 - Raw cap value from MOS device is 6.1 fF/ μm^2 for 0.24 μm CMOS
- Q (i.e., amount of series resistance)
 - Maximized with minimum L (tradeoff with area efficiency)

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A Recently Popular Approach – The MOS Varactor

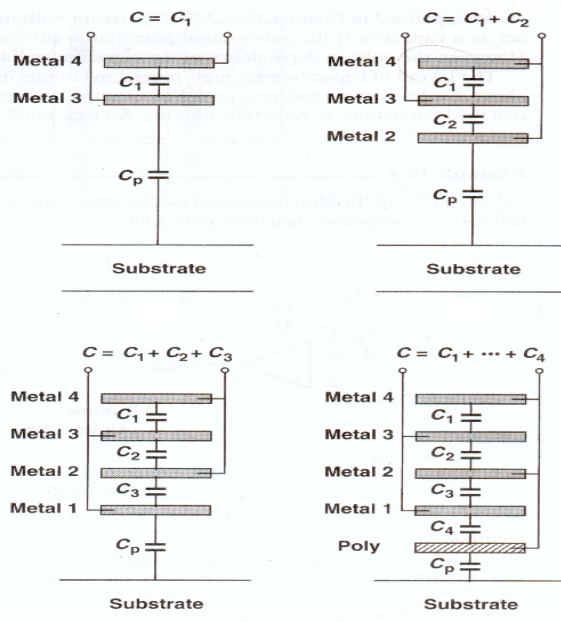
- Consists of a MOS transistor (NMOS or PMOS) with drain and source connected together
 - Abrupt shift in capacitance as inversion channel forms
- Advantage – easily integrated in CMOS
- Disadvantage – Q is relatively low in the transition region
 - Note that large signal is applied to varactor – transition region will be swept across each VCO cycle



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Vertical Metal Capacitors

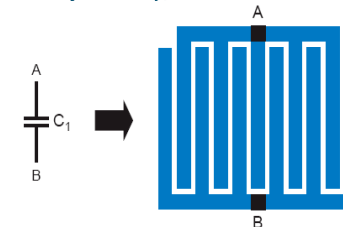


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Lateral Metal Capacitors

- Lateral metal capacitors offer high Q and reasonably large capacitance per unit area
 - Stack many levels of metal on top of each other (best layers are the top ones), via them at maximum density

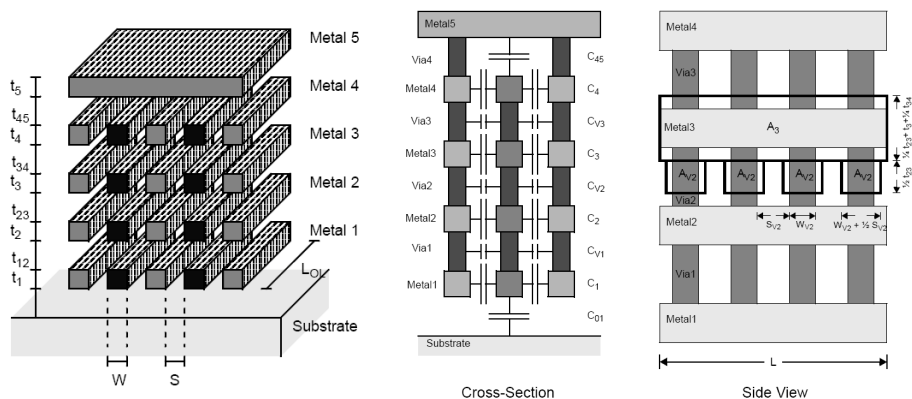


- Accuracy often better than $\pm 10\%$
- Parasitic side cap is symmetric, less than 10% of cap value
- Example: $C_T = 1.5 \text{ fF}/\mu\text{m}^2$ for 0.24 μm process with 7 metals, $L_{\min} = W_{\min} = 0.24\mu\text{m}$, $t_{\text{metal}} = 0.53\mu\text{m}$
 - See "Capacity Limits and Matching Properties of Integrated Capacitors", Aparicio et. al., JSSC, Mar 2002

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Vertical Mesh Metal Capacitors



$$C_{mesh} = 2(C_1 + C_{V1} + C_2 + C_{V2} + C_3 + C_{V3} + C_4) + C_{45}$$

$$C_{par} = C_{01}$$

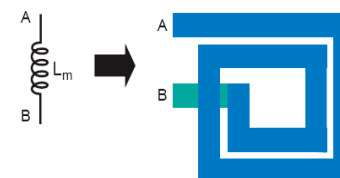
K.T. Christensen, "Low Power RF Filtering for CMOS Transceivers", Ph.D. Denmark Technical University, 2001, http://phd.dtv.dk/2001/oersted/k_t_christensen.pdf

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Spiral Inductors

- Create integrated inductor using spiral shape on top level metals (may also want a patterned ground shield)



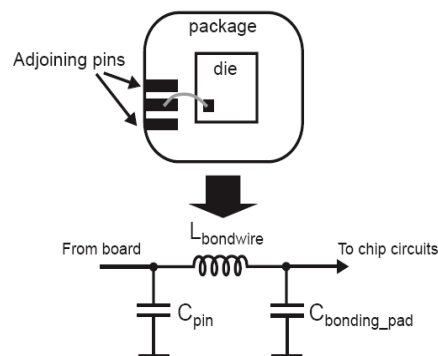
- Key parameters are Q (< 10), L (1-10 nH), self resonant freq.
- Usually implemented in top metal layers to minimize series resistance, coupling to substrate
- Design using Mohan et. al, "Simple, Accurate Expressions for Planar Spiral Inductances, JSSC, Oct, 1999, pp 1419-1424
- Verify inductor parameters (L, Q, etc.) using ASITIC <http://formosa.eecs.berkeley.edu/~niknejad/asitic.html>

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Bondwire Inductors

- Used to bond from the package to die
 - Can be used to advantage



- Key parameters
 - Inductance (≈ 1 nH/mm – usually achieve 1-5 nH)
 - Q (much higher than spiral inductors – typically > 40)

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Other Types of Resonators

- Quartz crystal
 - Very high Q, and very accurate and stable resonant frequency
 - Confined to low frequencies (< 200 MHz)
 - Non-integrated
 - Used to create low noise, accurate, "reference" oscillators
- SAW devices
 - High frequency, but poor accuracy (for resonant frequency)
- MEMS devices
 - Cantilever beams – promise high Q, but non-tunable and haven't made it to the GHz range, yet, for resonant frequency
 - FBAR – Q > 1000, but non-tunable and poor accuracy
 - Other devices are on the way!

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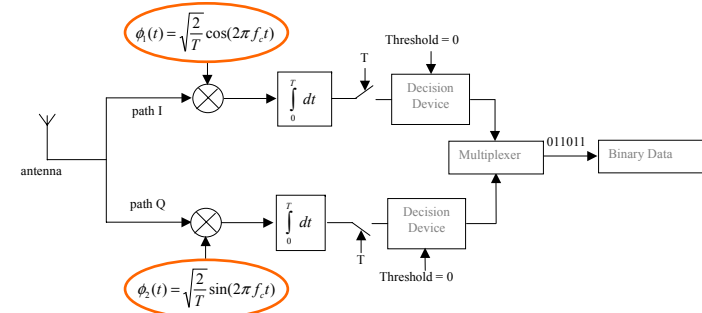
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Lecture II

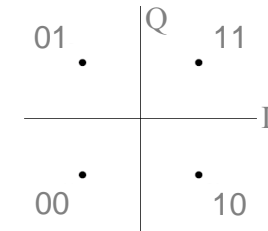
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QPSK Receiver



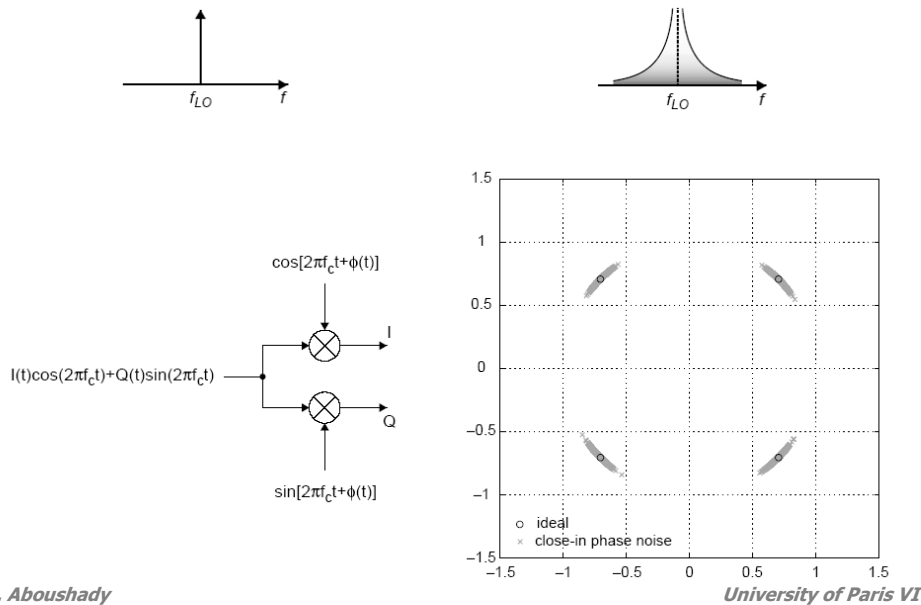
QPSK Constellation Diagram



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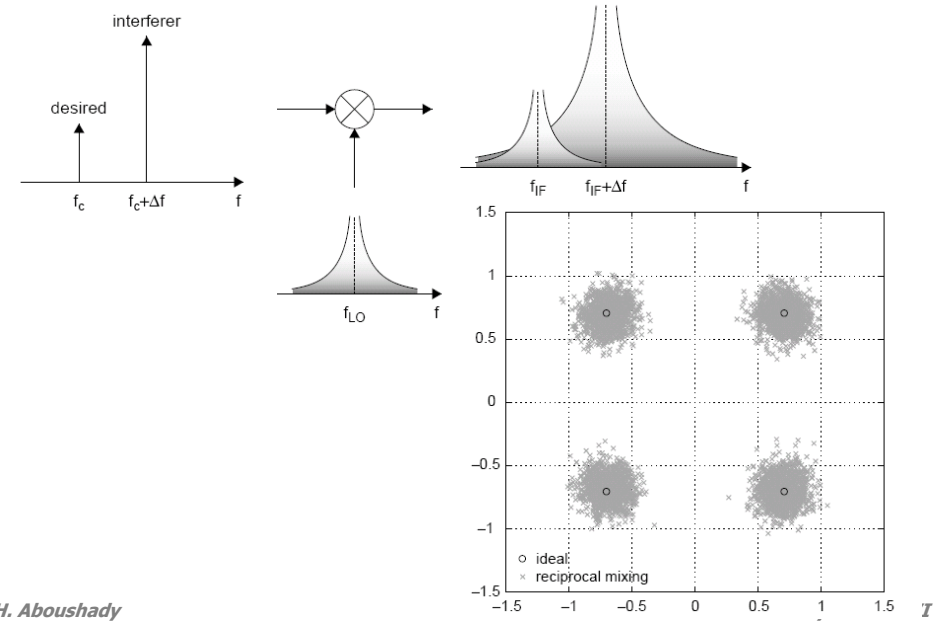
Local Oscillator Phase Noise



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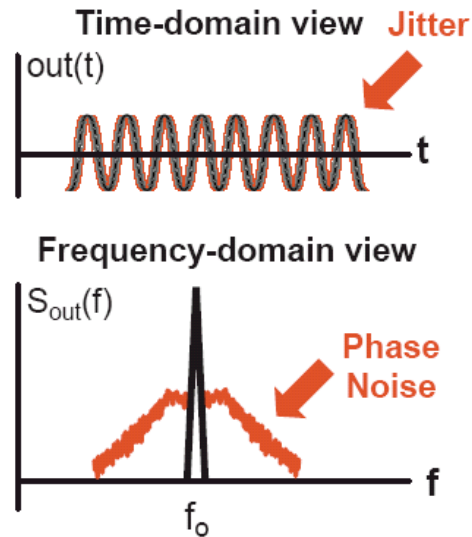
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Reciprocal Mixing



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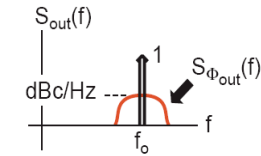
Phase Noise in Oscillators



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Measurement of Phase Noise in dBc/Hz



Definition of L(f)

$$L(f) = 10 \log \left(\frac{\text{Spectral density of noise}}{\text{Power of carrier}} \right)$$

- Units are dBc/Hz

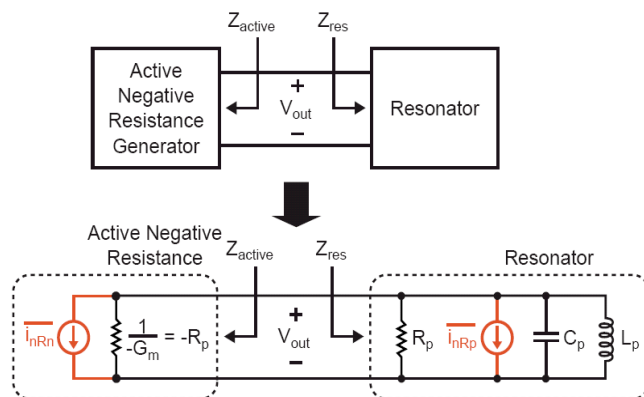
For this case

$$L(f) = 10 \log \left(\frac{S_{\Phi_{out}}(f)}{1} \right) = 10 \log(S_{\Phi_{out}}(f))$$

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Calculation of Intrinsic Phase Noise in Oscillators

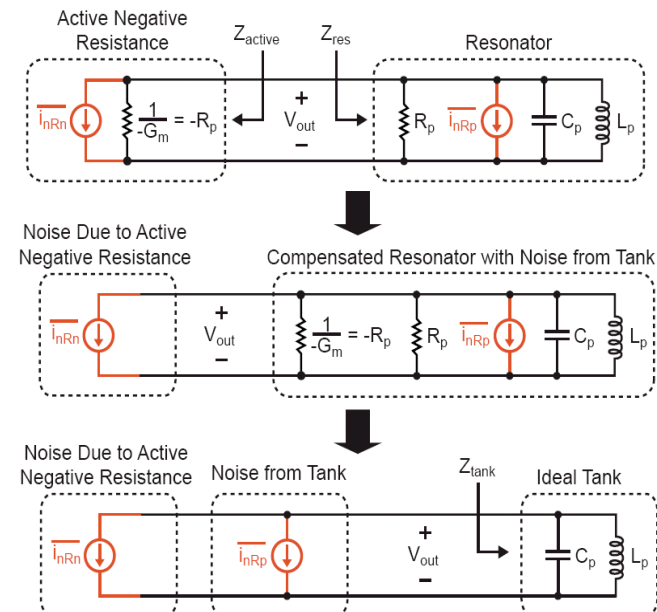


- Noise sources in oscillators are put in two categories
 - Noise due to tank loss
 - Noise due to active negative resistance
- We want to determine how these noise sources influence the phase noise of the oscillator

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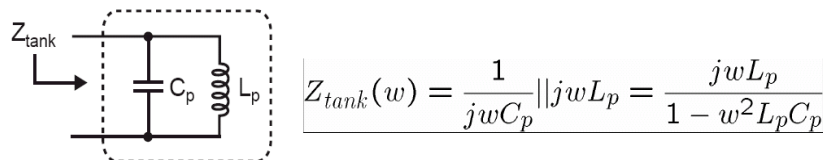
Equivalent Model for Noise Calculations



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Calculate Impedance Across Ideal LC Tank Circuit



- Calculate input impedance about resonance

Consider $w = w_o + \Delta w$, where $w_o = \frac{1}{\sqrt{L_p C_p}}$

$$Z_{tank}(\Delta w) = \frac{j(w_o + \Delta w)L_p}{1 - (w_o + \Delta w)^2 L_p C_p}$$

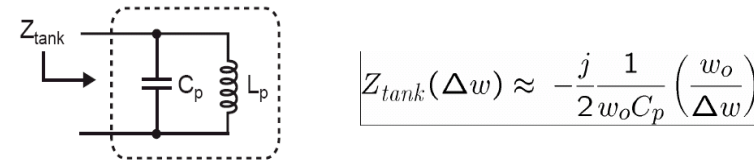
$$\approx \frac{j(w_o + \Delta w)L_p}{\underbrace{1 - w_o^2 L_p C_p}_{=0} - 2\Delta w(w_o L_p C_p) - \underbrace{\Delta w^2 L_p C_p}_{\text{negligible}}} \approx \frac{j(w_o + \Delta w)L_p}{-2\Delta w(w_o L_p C_p)}$$

$$\Rightarrow Z_{tank}(\Delta w) \approx \frac{jw_o L_p}{-2\Delta w(w_o L_p C_p)} = \boxed{-\frac{j}{2} \frac{1}{w_o C_p} \left(\frac{w_o}{\Delta w} \right)}$$

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A Convenient Parameterization of LC Tank Impedance



- Actual tank has loss that is modeled with R_p

- Define Q according to actual tank

$$Q = R_p w_o C_p \Rightarrow \frac{1}{w_o C_p} = \frac{R_p}{Q}$$

- Parameterize ideal tank impedance in terms of Q of actual tank

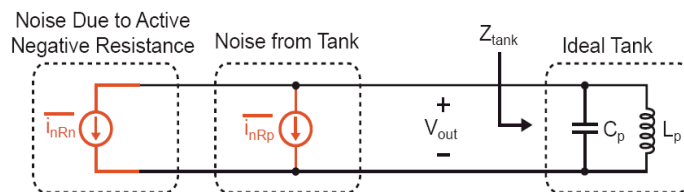
$$Z_{tank}(\Delta w) \approx -\frac{j R_p}{2 Q} \left(\frac{w_o}{\Delta w} \right)$$

$$\Rightarrow |Z_{tank}(\Delta f)|^2 \approx \boxed{\left(\frac{R_p f_o}{2Q \Delta f} \right)^2}$$

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Overall Noise Output Spectral Density



- Assume noise from active negative resistance element and tank are uncorrelated

$$\frac{v_{out}^2}{\Delta f} = \left(\frac{i_{nRp}^2}{\Delta f} + \frac{i_{nRn}^2}{\Delta f} \right) |Z_{tank}(\Delta f)|^2$$

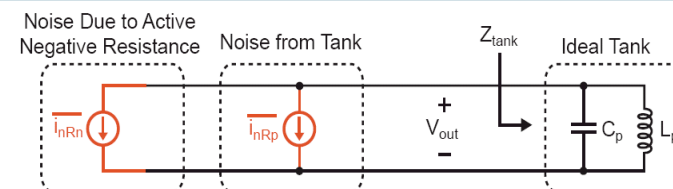
$$= \frac{i_{nRp}^2}{\Delta f} \left(1 + \frac{i_{nRn}^2}{\Delta f} / \frac{i_{nRp}^2}{\Delta f} \right) |Z_{tank}(\Delta f)|^2$$

- Note that the above expression represents total noise that impacts both amplitude and phase of oscillator output

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Parameterize Noise Output Spectral Density



- From previous slide

$$\frac{v_{out}^2}{\Delta f} = \frac{i_{nRp}^2}{\Delta f} \left(1 + \frac{i_{nRn}^2}{\Delta f} / \frac{i_{nRp}^2}{\Delta f} \right) |Z_{tank}(\Delta f)|^2$$

$F(\Delta f)$

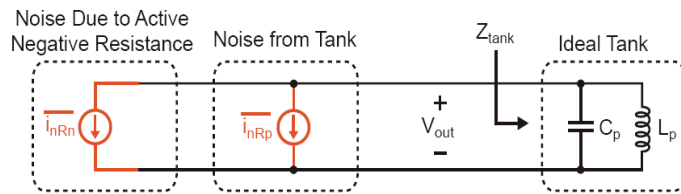
- $F(\Delta f)$ is defined as

$$F(\Delta f) = \frac{\text{total noise in tank at frequency } \Delta f}{\text{noise in tank due to tank loss at frequency } \Delta f}$$

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Fill in Expressions



- Noise from tank is due to resistor R_p

$$\frac{\overline{i_{nRp}^2}}{\Delta f} = 4kT \frac{1}{R_p} \quad (\text{single-sided spectrum})$$

- $Z_{\text{tank}}(\Delta f)$ found previously

$$|Z_{\text{tank}}(\Delta f)|^2 \approx \left(\frac{R_p f_o}{2Q \Delta f} \right)^2$$

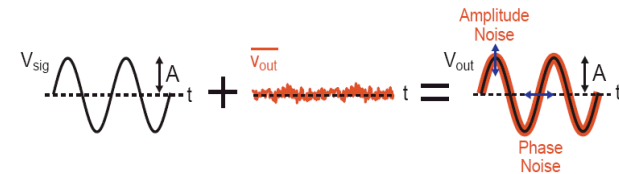
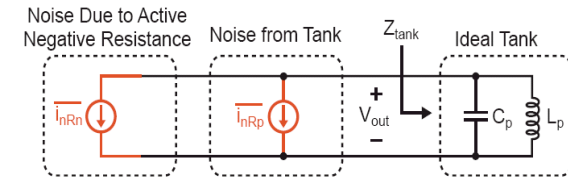
- Output noise spectral density expression (single-sided)

$$\frac{\overline{v_{out}^2}}{\Delta f} = 4kT \frac{1}{R_p} F(\Delta f) \left(\frac{R_p f_o}{2Q \Delta f} \right)^2 = 4kTF(\Delta f)R_p \left(\frac{1}{2Q \Delta f} f_o \right)^2$$

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Separation into Amplitude and Phase Noise



- Equipartition theorem states that noise impact splits evenly between amplitude and phase for V_{sig} being a sine wave

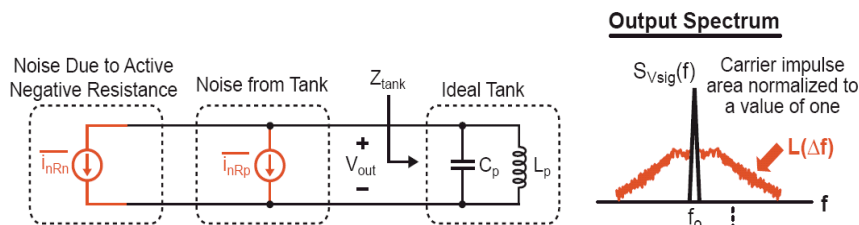
- Amplitude variations suppressed by feedback in oscillator

$$\Rightarrow \frac{\overline{v_{out}^2}}{\Delta f} \Big|_{\text{phase}} = 2kTF(\Delta f)R_p \left(\frac{1}{2Q \Delta f} f_o \right)^2 \quad (\text{single-sided})$$

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Output Phase Noise Spectrum (Leeson's Formula)



$$L(\Delta f) = 10 \log \left(\frac{\text{Spectral density of noise}}{\text{Power of carrier}} \right)$$

- All power calculations are referenced to the tank loss resistance, R_p

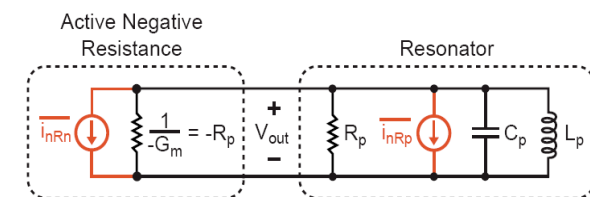
$$P_{\text{sig}} = \frac{V_{\text{sig,rms}}^2}{R_p} = \frac{(A/\sqrt{2})^2}{R_p}, \quad S_{\text{noise}}(\Delta f) = \frac{1}{R_p} \frac{\overline{v_{out}^2}}{\Delta f}$$

$$L(\Delta f) = 10 \log \left(\frac{S_{\text{noise}}(\Delta f)}{P_{\text{sig}}} \right) = 10 \log \left(\frac{2kTF(\Delta f)}{P_{\text{sig}}} \left(\frac{1}{2Q \Delta f} f_o \right)^2 \right)$$

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Example: Active Noise Same as Tank Noise

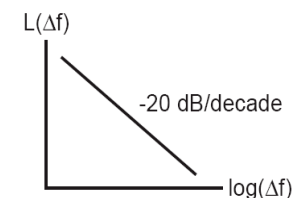


- Noise factor for oscillator in this case is

$$F(\Delta f) = 1 + \frac{\overline{i_{nRn}^2}}{\Delta f} / \frac{\overline{i_{nRp}^2}}{\Delta f} = 2$$

- Resulting phase noise

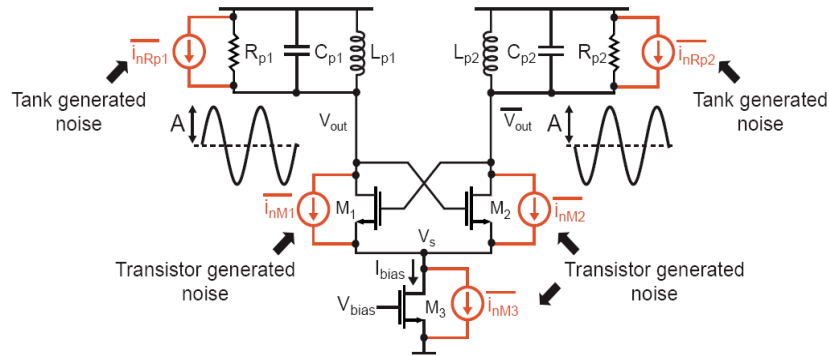
$$L(\Delta f) = 10 \log \left(\frac{4kT}{P_{\text{sig}}} \left(\frac{1}{2Q \Delta f} f_o \right)^2 \right)$$



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The Actual Situation is Much More Complicated

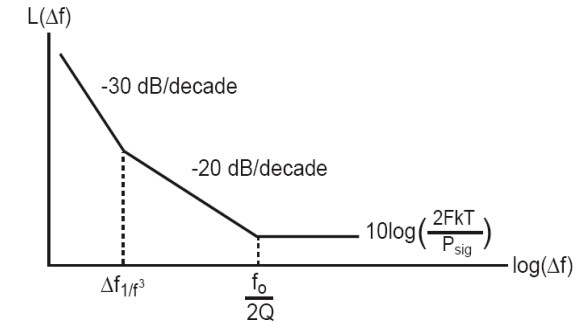


- Impact of tank generated noise easy to assess
- Impact of transistor generated noise is complicated
 - Noise from M_1 and M_2 is modulated on and off
 - Noise from M_3 is modulated before influencing V_{out}
 - Transistors have $1/f$ noise
- Also, transistors can degrade Q of tank

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Phase Noise of A Practical Oscillator

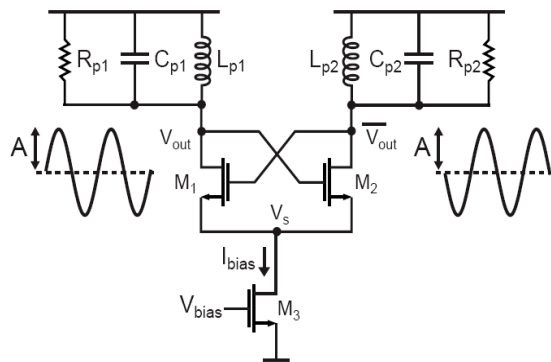


- Phase noise drops at -20 dB/decade over a wide frequency range, but deviates from this at:
 - Low frequencies – slope increases (often -30 dB/decade)
 - High frequencies – slope flattens out (oscillator tank does not filter all noise sources)
- Frequency breakpoints and magnitude scaling are not readily predicted by the analysis approach taken so far

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Approach for Negative Resistance Oscillator



- Recall Leeson's formula

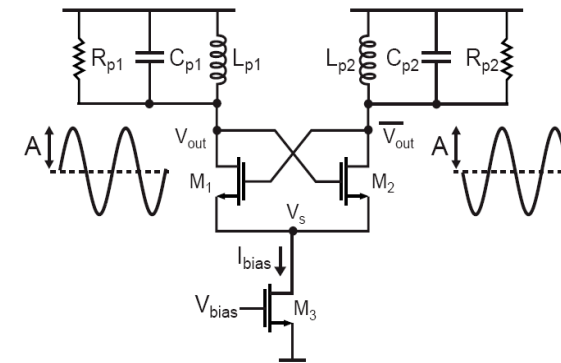
$$L(\Delta f) = 10 \log \left(\frac{2kTF(\Delta f)}{P_{sig}} \left(\frac{1}{2Q\Delta f} \right)^2 \right)$$

- Key question: how do you determine $F(\Delta f)$?

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$F(\Delta f)$ Has Been Determined for This Topology



- Rael et. al. have come up with a closed form expression for $F(\Delta f)$ for the above topology
- In the region where phase noise falls at -20 dB/dec:

$$F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi A} + \frac{4}{9} \gamma g_{do, M3} R_p \quad (R_p = R_{p1} = R_{p2})$$

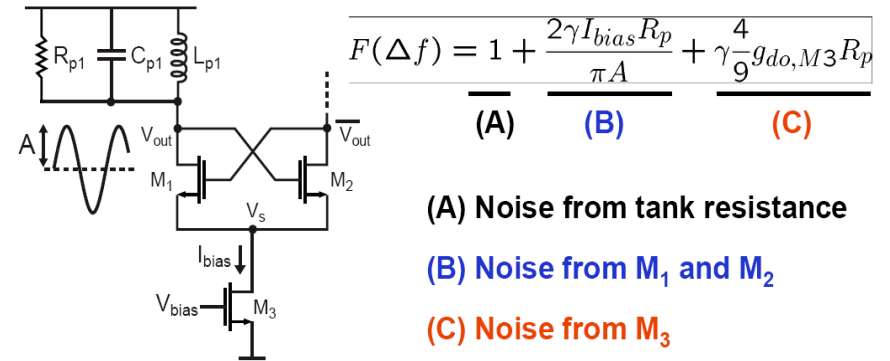
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References to Rael Work

- Phase noise analysis
 - J.J. Rael and A.A. Abidi, "Physical Processes of Phase Noise in Differential LC Oscillators", Custom Integrated Circuits Conference, 2000, pp 569-572
- Implementation
 - Emad Hegazi et. al., "A Filtering Technique to Lower LC Oscillator Phase Noise", JSSC, Dec 2001, pp 1921-1930

Designing for Minimum Phase Noise



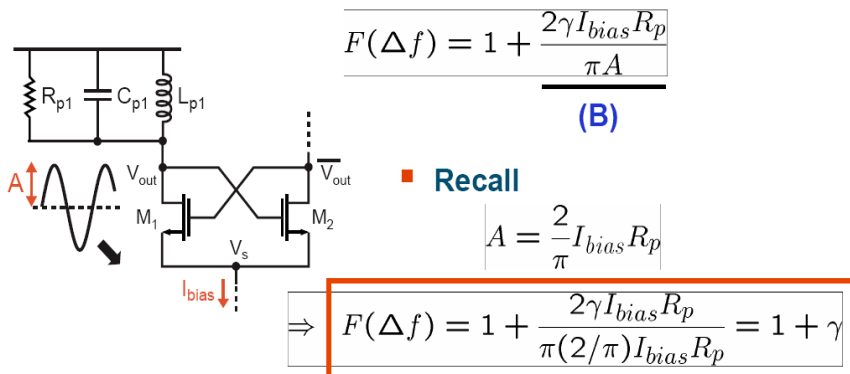
$$F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi A} + \gamma \frac{4}{9} g_{do, M3} R_p$$

(A) (B) (C)

- (A) Noise from tank resistance
- (B) Noise from \$M_1\$ and \$M_2\$
- (C) Noise from \$M_3\$

- To achieve minimum phase noise, we'd like to minimize \$F(\Delta f)\$
- The above formulation provides insight of how to do this
 - Key observation: (C) is often quite significant

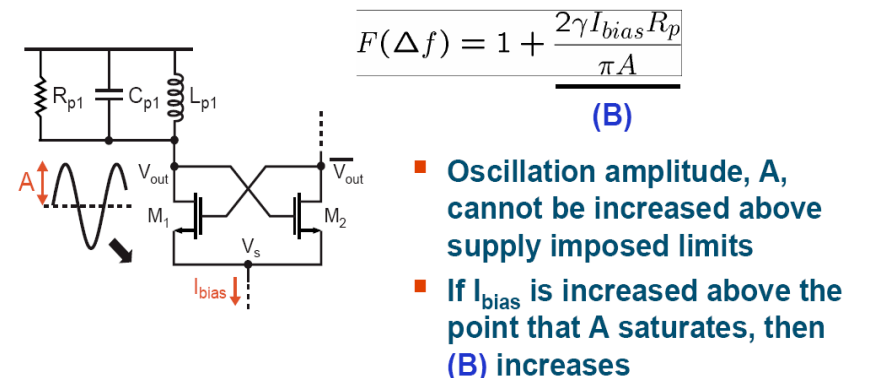
Minimization of Component (B) in \$F(\Delta f)\$



- So, it would seem that \$I_{bias}\$ has no effect!
 - Not true – want to maximize \$A\$ (i.e. \$P_{sig}\$) to get best phase noise, as seen by:

$$L(\Delta f) = 10 \log \left(\frac{2kTF(\Delta f)}{P_{sig}} \left(\frac{1}{2Q\Delta f} \right)^2 \right)$$

Current-Limited Versus Voltage-Limited Regimes



- Current-limited regime: amplitude given by $A = \frac{2}{\pi} I_{bias} R_p$
- Voltage-limited regime: amplitude saturated

Best phase noise achieved at boundary between these regimes!