Linearity and Noise Issues

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References

- B. Razavi, "RF Microelectronics", Prentice Hall, 1997.
- J. Rogers and C. Plett, "Radio Frequency Integrated Circuit Design", Artech, 2003.
- M. Perrott, "High Speed Communication Circuits and Systems", M.I.T.OpenCourseWare, http://ocw.mit.edu/, Massachusetts Institute of Technology, 2003.

Units in Microwave and RF Design

In low frequency analog circuits, infinite or zero impedance is allowed, power levels are meaningless, so voltages and currents are usually chosen to describe the signal levels.

Power is used to describe signals, noise, or distortion with the typical unit of measure being decibels above 1 milliwatt (dBm).

$$P_{dBm} = 10\log_{10}\left(\frac{P_{WATT}}{1\,mW}\right)$$

$$P_{WATT} = \frac{v_{rms}^2}{R} \quad , \quad v_{rms} = \frac{v_{pp}}{2\sqrt{2}}$$

Unless otherwise stated, R=50Ω.

Example:

v_{pp}	v_{rms}	P_{WATT}	P_{dBm}
$1\,nV$	$0.3536\,nV$	$2.5E-21$	-176

Analog vs RF Circuit Design

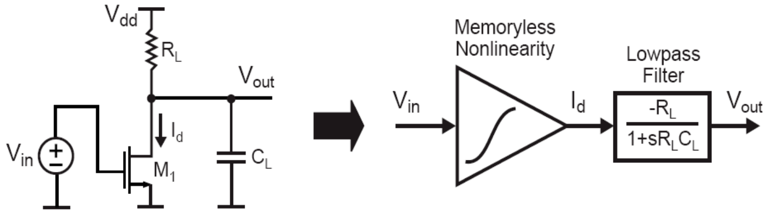
Parameter	Analog Design (most often used on chip)	Microwave Design (most often used at chip boundaries and pins)
Impedance	$Z_{in} \Rightarrow \infty$ $Z_{out} \Rightarrow 0$	$Z_{in} \Rightarrow 50\Omega$ $Z_{out} \Rightarrow 50\Omega$
Signals	Voltage, current, often peak or peak-to-peak	Power, often dBm
Noise	$nV/\sqrt{Hz}$	Noise factor *F*, noise figure NF
Nonlinearity	Harmonic distortion, intermodulation, clipping	Third-order intercept point IP3 1-dB compression

Overview

- Non-linearities
  - 1-dB compression point
  - Intermodulation distortion
  - 3rd intercept point
- Noise
  - Noise models
  - Noise figure

Nonlinearities in Amplifiers

- **We can generally break up an amplifier into the cascade of a memoryless nonlinearity and an input and/or output transfer function**



- **Impact of nonlinearities with sine wave input**
  - **Causes harmonic distortion (i.e., creation of harmonics)**
- **Impact of nonlinearities with several sine wave inputs**
  - **Causes harmonic distortion for each input AND intermodulation products**

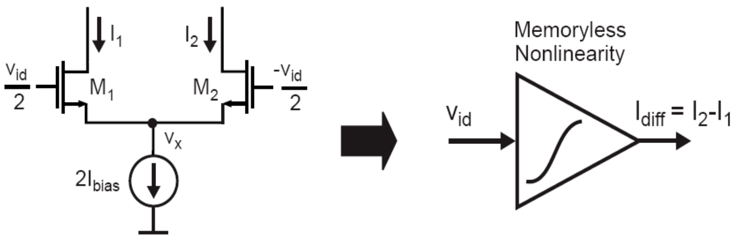
Non-linearity

$$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots,$$

$$y(t) \approx \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t).$$

$$\begin{aligned} y(t) &= \alpha_1 A \cos \omega t + \alpha_2 A^2 \cos^2 \omega t + \alpha_3 A^3 \cos^3 \omega t \\ &= \alpha_1 A \cos \omega t + \frac{\alpha_2 A^2}{2}(1 + \cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3 \cos \omega t + \cos 3\omega t) \\ &= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right) \cos \omega t + \frac{\alpha_2 A^2}{2} \cos 2\omega t + \frac{\alpha_3 A^3}{4} \cos 3\omega t. \end{aligned}$$

Impact of Differential Amplifiers on Nonlinearity



- **Assume v_x is approximately incremental ground**

$$I_{diff} = c_o + c_1 \frac{v_{id}}{2} + c_2 \left(\frac{v_{id}}{2}\right)^2 + c_3 \left(\frac{v_{id}}{2}\right)^3$$

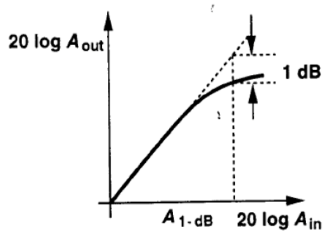
$$- \left(c_o + c_1 \frac{-v_{id}}{2} + c_2 \left(\frac{-v_{id}}{2}\right)^2 + c_3 \left(\frac{-v_{id}}{2}\right)^3\right)$$

$$\Rightarrow \quad I_{diff} = c_1 v_{id} + \frac{c_3}{4} v_{id}^3$$

- **Second order term removed and IIP3 increased!**

M.H. Perrott *MIT OCW*

1-dB compression point (CP_1)



$$20\log|\alpha_1 + \frac{3}{4}\alpha_3 A_{1\text{-dB}}^2| = 20\log|\alpha_1| - 1\text{ dB}.$$

$$A_{1\text{-dB}} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

(α_3 is negative)

Desensitization and blocking

$$x(t) = A_1\cos\omega_1 t + A_2\cos\omega_2 t.$$

$$y(t) = \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right)\cos\omega_1 t + \cdots$$

if $A_1 << A_2$:

$$y(t) = \left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\right)A_1\cos\omega_1 t + \cdots.$$

Gain falls with growing A_2

(α_3 is negative)

IMD

Good IMD behavior is crucial in all modern radio systems

$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t.$$

$$y(t) = \alpha_1(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t) + \alpha_2(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^2 + \alpha_3(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^3. \quad (2.21)$$

More IMD

Fundamental Components:

$$\omega = \omega_1, \omega_2 : \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right) \cos \omega_1 t + \left(\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_2 A_1^2\right) \cos \omega_2 t.$$

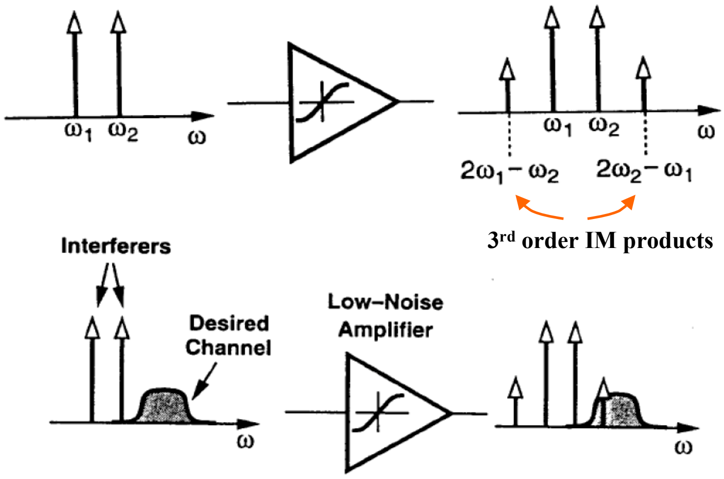
Inter-Modulation Components:

$$\omega = \omega_1 \pm \omega_2 : \alpha_2 A_1 A_2 \cos(\omega_1 + \omega_2)t + \alpha_2 A_1 A_2 \cos(\omega_1 - \omega_2)t$$

$$= 2\omega_1 \pm \omega_2 : \frac{3\alpha_3 A_1^2 A_2}{4} \cos(2\omega_1 + \omega_2)t + \frac{3\alpha_3 A_1^2 A_2}{4} \cos(2\omega_1 - \omega_2)t$$

$$= 2\omega_2 \pm \omega_1 : \frac{3\alpha_3 A_2^2 A_1}{4} \cos(2\omega_2 + \omega_1)t + \frac{3\alpha_3 A_2^2 A_1}{4} \cos(2\omega_2 - \omega_1)t$$

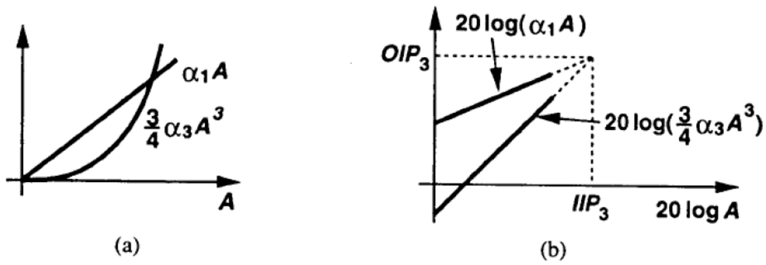
Inter-modulation distortion (IMD)



Corruption of a signal due to intermodulation between two interferers.

3rd intercept point (IP_3)

Most common way of describing the IMD performance of a system



Growth of output components in an intermodulation test.

Expression of IP_3

$$x(t) = A\cos\omega_1 t + A\cos\omega_2 t.$$

$$y(t) = \left(\alpha_1 + \frac{9}{4}\alpha_3 A^2\right) A\cos\omega_1 t + \left(\alpha_1 + \frac{9}{4}\alpha_3 A^2\right) A\cos\omega_2 t$$
$$+ \frac{3}{4}\alpha_3 A^3 \cos(2\omega_1 - \omega_2)t + \frac{3}{4}\alpha_3 A^3 \cos(2\omega_2 - \omega_1)t + \cdots.$$

$$|\alpha_1| A_{IP3} = \frac{3}{4}|\alpha_3| A_{IP3}^3.$$

$$A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

Measurement of IP_3

$A_{\omega1,\omega2}$: The amplitude of the output components at ω1 and ω2
A_{IM3} : The amplitude of the IM3 products

$$\frac{A_{\omega1,\omega2}}{A_{IM3}} \approx \frac{|\alpha_1| A_{in}}{3|\alpha_3| A_{in}^3/4} = \frac{4|\alpha_1|}{3|\alpha_3|}\frac{1}{A_{in}^2},$$

and

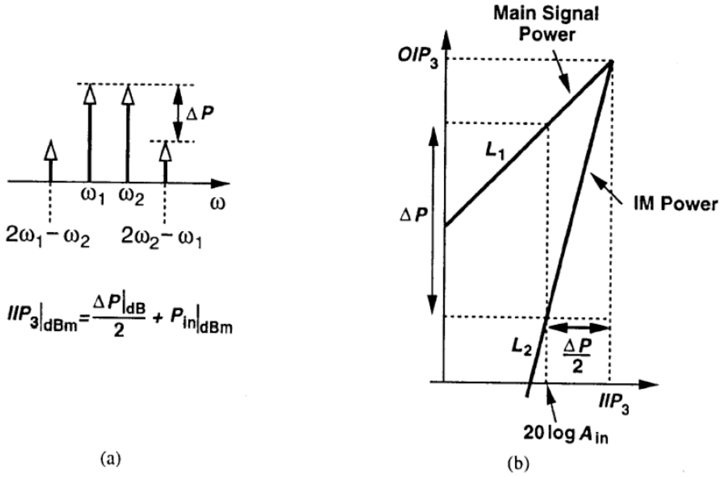
$$A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

$$\frac{A_{\omega1,\omega2}}{A_{IM3}} = \frac{A_{IP3}^2}{A_{in}^2}.$$

$$20\log A_{\omega1,\omega2} - 20\log A_{IM3} = 20\log A_{IP3}^2 - 20\log A_{in}^2,$$

$$20\log A_{IP3} = \frac{1}{2}(20\log A_{\omega1,\omega2} - 20\log A_{IM3}) + 20\log A_{in}.$$

Measurement of IP_3



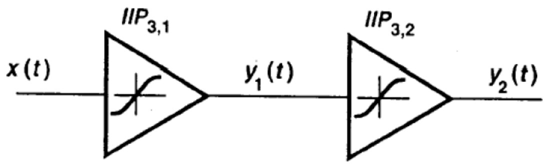
(a) Calculation of IP_3 without extrapolation, (b) graphical interpretation of (a).

CP1 and IP3

$$A_{1\text{-dB}} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}. \quad \text{and} \quad A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

$$\frac{A_{1\text{-dB}}}{A_{IP3}} = \frac{\sqrt{0.145}}{\sqrt{4/3}} \approx -9.6\,\text{dB}.$$

Cascaded nonlinear stages



$$y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$
$$y_2(t) = \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 y_1^3(t),$$

$$\begin{aligned} y_2(t) = &\ \beta_1[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)] \\ &+ \beta_2[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^2 \\ &+ \beta_3[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^3. \end{aligned}$$

Cascaded nonlinear stages

Considering only the 1st and 3rd order terms:

$$y_2(t) = \alpha_1\beta_1 x(t) + (\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3)x^3(t) + \cdots.$$

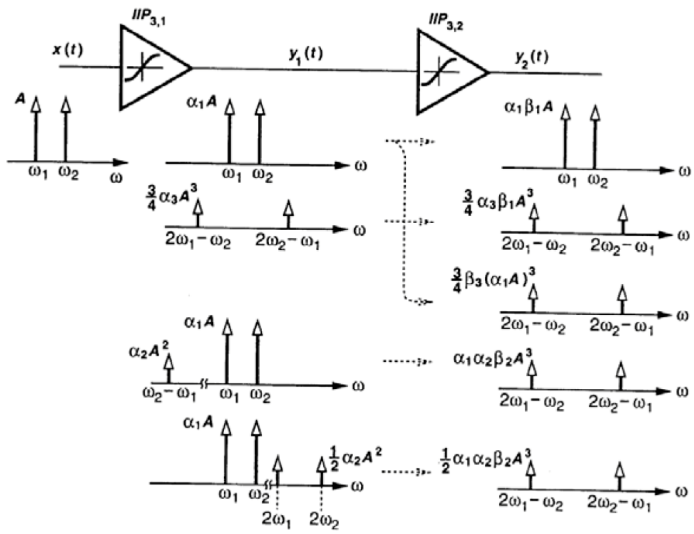
$$A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}. \Rightarrow A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1\beta_1}{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3}\right|}.$$

As a worst case estimate, we add the absolute values of the 3 terms in the denominator :

$$\begin{aligned} \frac{1}{A_{IP3}^2} &= \frac{3}{4}\frac{|\alpha_3\beta_1| + |2\alpha_1\alpha_2\beta_2| + |\alpha_1^3\beta_3|}{|\alpha_1\beta_1|} \\ &= \frac{1}{A_{IP3,1}^2} + \frac{3\alpha_2\beta_2}{2\beta_1} + \frac{\alpha_1^2}{A_{IP3,2}^2}, \end{aligned}$$

- As $\alpha 1$ increases, the overall IP3 decreases ⇨ :-(
- Higher 1st stage Gain ⇨ Higher input for 2nd stage ⇨ Greater IM3

Cascaded nonlinear stages



Cascaded nonlinear stages

$$\frac{1}{A_{IP3}^2} = \frac{3}{4}\frac{|\alpha_3\beta_1| + |2\alpha_1\alpha_2\beta_2| + |\alpha_1^3\beta_3|}{|\alpha_1\beta_1|}$$

$$= \frac{1}{A_{IP3,1}^2} + \frac{3\alpha_2\beta_2}{2\beta_1} + \frac{\alpha_1^2}{A_{IP3,2}^2},$$

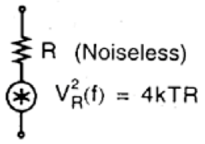
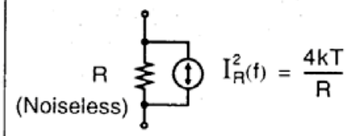
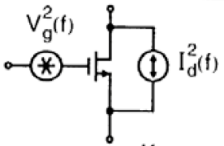
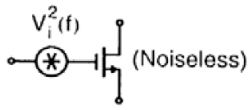
becomes (with interstage filtering)

$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2}.$$

$$\boxed{\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2} + \frac{\alpha_1^2\beta_1^2}{A_{IP3,3}^2} + \cdots,}$$

Non-linearity of the latter stages becomes increasingly more critical !

Noise

Element	Noise Models	
Resistor 	 $V_R^2(f) = 4kTR$	 $I_R^2(f) = \frac{4kT}{R}$
MOSFET  (Active region)	 $V_g^2(f) = \frac{K}{WLC_{ox}f}$ $I_d^2(f) = 4kT\left(\frac{2}{3}\right)g_m$	 $V_i^2(f) = 4kT\left(\frac{2}{3}\right)\frac{1}{g_m} + \frac{K}{WLC_{ox}f}$ Simplified model for low and moderate frequencies

Thermal Noise

Noise Spectral Density:

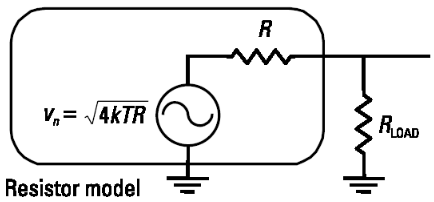
$$N_{resistor} = 4kTR \quad (V^2 / Hz)$$

The bandwidth of interest:

$$\Delta f$$

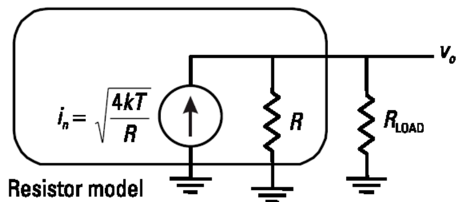
Mean square noise voltage:

$$v_n^2 = 4kTR\Delta f$$

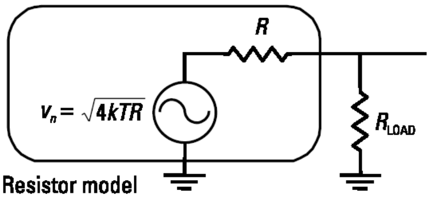


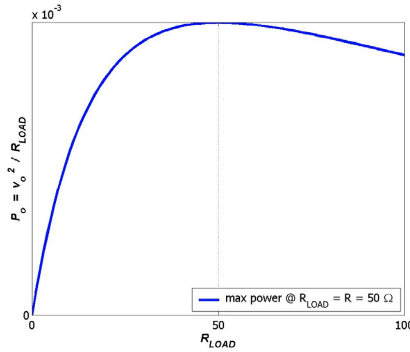
Mean square noise current:

$$i_n^2 = \frac{4kT\Delta f}{R}$$



Available Noise Power





Output Power Spectral Density:

$$P_0 = \frac{v_0^2}{R_{LOAD}} = \frac{v_n^2\, R_{LOAD}}{(R + R_{LOAD})^2}$$

$$\Rightarrow \quad P_o = \frac{v_o^2}{R} = \frac{v_n^2}{4R} = kT \qquad (Watts\,/\,Hz)$$

Total Output Power:

$$\Rightarrow \quad P_{\text{out}} = kTB$$

Available Power from Antenna

The noise from an antenna can be modeled as a resistor:

$$P_{\text{available}} = kT = 4 \times 10^{-21} \text{ W/Hz}$$

$$\Rightarrow \quad P_{\text{available}} = 10 \log_{10}\left(\frac{4 \times 10^{-21}}{1 \times 10^{-3}}\right) = -174 \text{ dBm/Hz}$$

The minimum detectable signal (e.g. *B=200 kHz*):

$$\text{Noise floor} = kTB$$

$$= 4 \times 10^{-21} \times 200{,}000 = 8 \times 10^{-16}$$

Usually expressed in dBm

$$\text{Noise floor} = -174 \text{ dBm/Hz} + 10 \log_{10}(200{,}000) = -121 \text{ dBm}$$

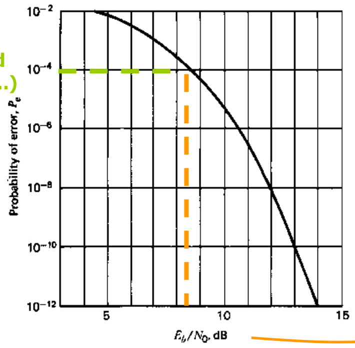
Receiver Sensitivity

If the signal has a power S: $$\text{SNR} = \frac{S}{\text{Noise floor}}$$

~~- If~~ the electronics add~~ed~~s ~~no~~ noise,
~~- If~~ the detector require~~d~~s a SNR of *8.5 dB*
⇨ Then a signal as low as -~~121~~ dBm could be detected.

"Receiver Sensitivity is the minimum detectable signal"

BER Determined by the communication standard (GSM, Bluetooth, WiFi, …)



Noise Figure

Noise from the electronics is described by noise factor F, which is a measure of how much the SNR is degraded through the system:

$$S_o = G \cdot S_i$$

where

S_i : Input Signal Power, S_0 : Output Signal Power, $G = \frac{S_0}{S_i}$: Power Gain.

Noise Factor: $$F = \frac{\text{SNR}_i}{\text{SNR}_o} = \frac{S_i / N_{i\,(\text{source})}}{S_o / N_{o\,(\text{total})}} = \frac{S_i / N_{i\,(\text{source})}}{(S_i \cdot G) / N_{o\,(\text{total})}} = \frac{N_{o\,(\text{total})}}{G \cdot N_{i\,(\text{source})}}$$

$N_{o\,(\text{total})}$ is the total noise at the output

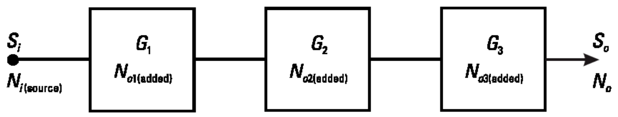
$N_{o\,(\text{source})}$ is the noise at the output originating at the source,

$$N_{o\,(\text{total})} = N_{o\,(\text{source})} + N_{o\,(\text{added})}$$

$$F = \frac{N_{o\,(\text{total})}}{G \cdot N_{i\,(\text{source})}} = \frac{N_{o\,(\text{total})}}{N_{o\,(\text{source})}} = \frac{N_{o\,(\text{source})} + N_{o\,(\text{added})}}{N_{o\,(\text{source})}} = 1 + \frac{N_{o\,(\text{added})}}{N_{o\,(\text{source})}}$$

Noise Figure: $$\text{NF} = 10 \log_{10} F$$

Noise Figure of components in Series



The output signal S_o is given by $S_o = S_i \cdot G_i \cdot G_2 \cdot G_3$

The input noise is $N_{i\,(\text{source})} = kT$

The total output noise is $N_{o\,(\text{total})} = N_{i\,(\text{source})} G_1 G_2 G_3 + N_{o1\,(\text{added})} G_2 G_3 + N_{o2(\text{added})} G_3 + N_{o3(\text{added})}$

The output noise due to the source is $N_{o\,(\text{source})} = N_{i\,(\text{source})} G_1 G_2 G_3$

Noise Factor:

$$F = \frac{N_{o\,(\text{total})}}{N_{o\,(\text{source})}} = 1 + \frac{N_{o1\,(\text{added})}}{N_{i\,(\text{source})} G_1} + \frac{N_{o2(\text{added})}}{N_{i\,(\text{source})} G_1 G_2} + \frac{N_{o3(\text{added})}}{N_{i\,(\text{source})} G_1 G_2 G_3}$$

Noise Figure of components in Series

Noise Factor:

$$F = \frac{N_{o\,(\text{total})}}{N_{o\,(\text{source})}} = 1 + \frac{N_{o1\,(\text{added})}}{N_{i\,(\text{source})} G_1} + \frac{N_{o2(\text{added})}}{N_{i\,(\text{source})} G_1 G_2} + \frac{N_{o3(\text{added})}}{N_{i\,(\text{source})} G_1 G_2 G_3}$$

$$\text{Since, } F_1 = 1 + \frac{N_{01(added)}}{N_{01(source)}} \Rightarrow F_1 - 1 = \frac{N_{01(added)}}{N_{i(source)} G_1}$$

$$\text{Similarly, } F_2 - 1 = \frac{N_{02(added)}}{N_{i(source)} G_2}$$

$$\text{and, } F_3 - 1 = \frac{N_{03(added)}}{N_{i(source)} G_3}$$

$$F = 1 + (F_1 - 1) + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2}$$

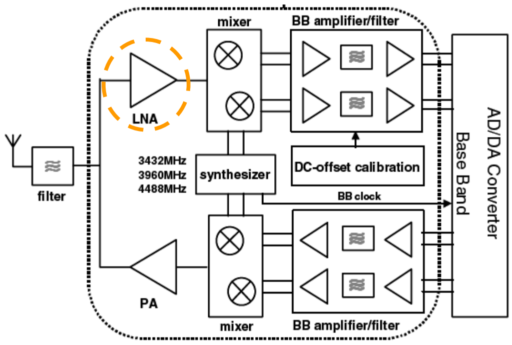
Friis Equation

Noise Factor:

$$F = 1 + (F_1 - 1) + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} + \ldots + \frac{(F_n - 1)}{G_1 G_2 \ldots G_{n-1}}$$

Gain in stage *n* reduces the effective noise figure of stage *n+1*

The 1st stage of RF receivers: LNA (Low Noise Amplifier)



Equivalent Noise Model

$$\text{Since, } F_1 - 1 = \frac{N_{01(added)}}{N_{i(source)} G_1} \quad \text{and, } N_{i1(added)} = \frac{N_{01(added)}}{G_1}$$

$$\text{we have: } N_{i1(added)} = (F_1 - 1) N_{i(source)}$$

$$N_{i(source)} = kT$$

$$\boxed{N_{i1(added)} = (F_1 - 1)\, kT}$$