Single Stage Amplifiers

- ***Basic Concepts***
- ***Common Source Stage***
- ***Source Follower***
- ***Common Gate Stage***
- ***Cascode Stage***

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References

- **B. Razavi, "Design of Analog CMOS Integrated Circuits", McGraw-Hill, 2001.**

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Basic Concepts I

- **Amplification is an essential function in most analog circuits !**
- **Why do we amplify a signal ?**
  - **The signal is too small to drive a load**
  - **To overcome the noise of a subsequent stage**
  - **Amplification plays a critical role in feedback systems**

In this lecture:
- **Low frequency behavior of single stage CMOS amplifiers:**
  - **Common Source, Common Gate, Source Follower, ...**
- **Large and small signal analysis.**
- **We begin with a simple model and gradually add 2nd order effects**

➡ **Understand basic building blocks for more complex systems.**

Approximation of a nonlinear system

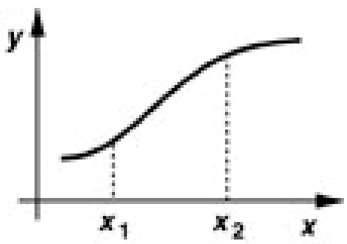
Input-Output Characteristic of a nonlinear system

$$y(t) \approx \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \ldots + \alpha_n x^n(t) \qquad x_1 \le x \le x_2$$

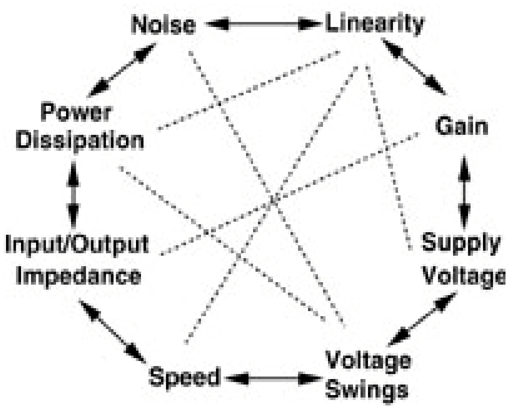
In a sufficiently narrow range:

$$y(t) \approx \alpha_0 + \alpha_1 x(t)$$

where α_0 can be considered the operating (bias) point and α_1 the small signal gain



Analog Design Octagon



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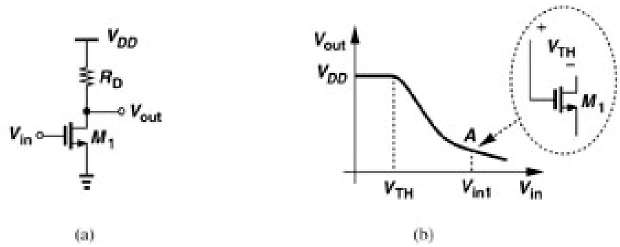
Common Source Stage with Resistive Load



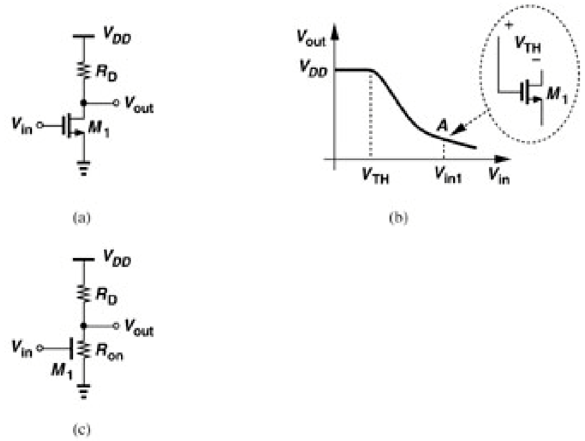
$$V_{out} = V_{DD} - R_D I_D$$

M1 in the saturation region: $$V_{out} = V_{DD} - R_D \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{in} - V_{TH})^2$$

M1 in limit of saturation: $$V_{in1} - V_{TH} = V_{DD} - R_D \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{in1} - V_{TH})^2$$

M1 in the linear region: $$V_{out} = V_{DD} - R_D \mu_n C_{ox} \frac{W}{L} \left[(V_{in} - V_{TH}) V_{out} - \frac{V_{out}^2}{2} \right]$$

Common Source Stage with Resistive Load



M1 in deep linear region:

$$V_{out} = V_{DD}\frac{R_{on}}{R_{on}+R_D} = \frac{V_{DD}}{1+\mu_n C_{ox}\frac{W}{L}R_D(V_{in}-V_{TH})}$$

Common Source Stage with Resistive Load

M1 in the saturation region:

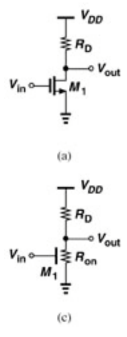
$$V_{out} = V_{DD} - R_D\frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{in}-V_{TH})^2$$

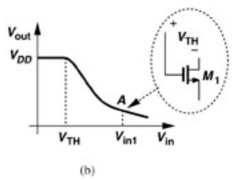
Small signal gain:

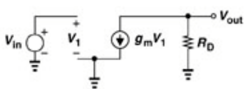
$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = -R_D\mu_n C_{ox}\frac{W}{L}(V_{in}-V_{TH})$$

$$= -g_m R_D$$

Same relation can be derived from the small signal equivalent circuit





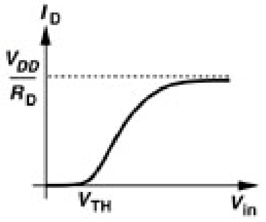


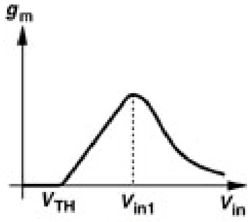
Small signal model for the saturation region

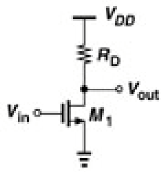
To minimize nonlinearity, the gain equation must be a weak function of signal dependent parameters such as g_m !

Example 1

Sketch I_D and g_m of M1 as a function of the V_{in}:







- **M1 in the saturation region:**

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{in} - V_{TH})^2$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})$$

- **M1 in the linear region:**

$$I_D = \mu_n C_{ox}\frac{W}{L}\left[(V_{in1} - V_{TH})V_{out} - \frac{V_{out}^2}{2}\right]$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\frac{W}{L}V_{out}$$

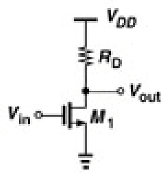
$$V_{out} = \frac{V_{DD}}{1 + \mu_n C_{ox}\frac{W}{L}R_D(V_{in} - V_{TH})}$$

Voltage Gain of a Common Source Stage

$$A_v = -g_m R_D$$

$$A_v = -\sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}\,\frac{V_{RD}}{I_D}$$

$$\boxed{A_v = -\sqrt{2\mu_n C_{ox}\frac{W}{L}}\,\frac{V_{RD}}{\sqrt{I_D}}}$$



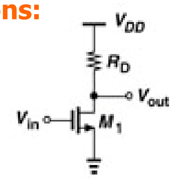
How to increase A_v ?

Trade-offs:

- **Increase W/L** ➡ **Greater device capacitances.**
- **Increase V_{RD}** ➡ **Limits V_{out} swing.**
- **Reduce I_D** ➡ **Greater Time Constant.**

Taking Channel Length Modulation into account

Calculating A_v starting from the Large Signal Equations:



$$V_{out} = V_{DD} - R_D \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{in} - V_{TH})^2 (1 + \lambda V_{out})$$

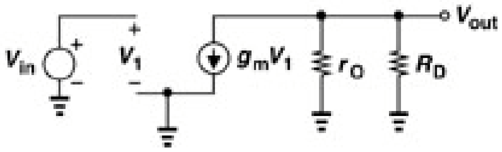
$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = -R_D \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})(1 + \lambda V_{out}) - R_D \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{in} - V_{TH})^2 \lambda \frac{\partial V_{out}}{\partial V_{in}}$$

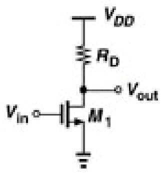
$$A_v = -R_D g_m - R_D I_D \lambda A_v$$

$$A_v = -\frac{R_D g_m}{1 + R_D \lambda I_D} \xrightarrow{\lambda I_D = 1/r_O} \boxed{A_v = -g_m \frac{r_O R_D}{r_O + R_D}}$$

Taking Channel Length Modulation into account

Calculating A_v starting from the Small Signal model:

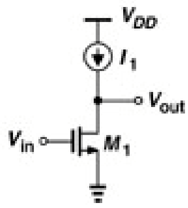




$$\left. \begin{aligned} g_m V_1 (r_O // R_D) &= -V_{out} \\ V_1 &= V_{in} \end{aligned} \right\} \boxed{A_v = \frac{V_{out}}{V_{in}} = -g_m (r_O // R_D)}$$

Example 2

**Assuming M1 biased in saturation,
calculate the small signal voltage gain :**



- **I_1 : Ideal current source ➡ Infinite Impedance**

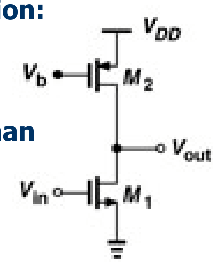
$$\boxed{A_v = -g_m r_O}$$

- **<u>Intrinsic gain of a transistor:</u>**
This quantity represents the maximum voltage gain that can be achieved using a single device.

$$I_{D1} = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{in} - V_{TH})^2(1 + \lambda V_{out}) = I_1$$

- **<u>Constant Current:</u>**
As V_{in} increases, V_{out} must decrease such that the product remains constant

CS Stage with Current-Source Load



- **Both transistors operate in the saturation region:**

$$\boxed{A_v = -g_m (r_{O1} // r_{O2})}$$

- **The output impedance and the minimum required VDS of M2 are less strongly coupled than the value and voltage drop of a resistor.**

$$|V_{DS2,\min}| = |V_{GS2} - V_{TH2}|$$

- **This value can be reduced to a few hundred millivolts by simply increasing the width of M2.**
- **If r_{O2} is not sufficiently high, the length and width of M2 can be increased to achieve a smaller λ while maintaining the same overdrive voltage.**
- **The penalty is the large capacitance introduced by M2 at the output node.**
- **Increasing L2 while keeping W2 constant increases r_{O2} and hence the voltage gain, but at the cost of higher |VDS2| required to maintain M2 in saturation**

CS with Source Degeneration

Large Signal model:

$$G_m = \frac{\partial I_D}{\partial V_{in}} = \frac{\partial I_D}{\partial V_{GS}} \frac{\partial V_{GS}}{\partial V_{in}}$$

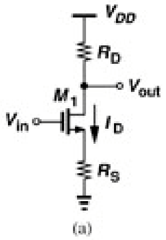
$$V_{GS} = V_{in} - I_D R_S$$

$$\frac{\partial V_{GS}}{\partial V_{in}} = 1 - \frac{\partial I_D}{\partial V_{in}} R_S$$

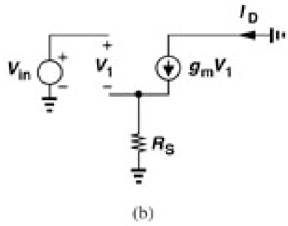
$$G_m = \frac{\partial I_D}{\partial V_{GS}} \left(1 - R_S \frac{\partial I_D}{\partial V_{in}}\right)$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}}$$

$$G_m = g_m (1 - R_S G_m)$$



Small Signal model:



$$G_m = \frac{I_D}{V_{in}} = \frac{g_m V_1}{V_1 + g_m V_1 R_S}$$

$$\boxed{G_m = \frac{g_m}{1 + g_m R_S}}$$

$$A_v = -G_m R_D$$

$$\boxed{G_m = \frac{g_m}{1 + g_m R_S}}$$

$$\boxed{A_v = -\frac{g_m R_D}{1 + g_m R_S}}$$

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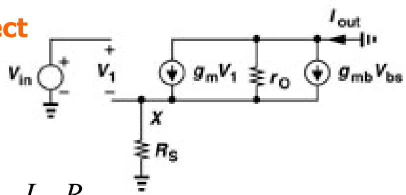
CS with Source Degeneration

$$\boxed{G_m = \frac{g_m}{1 + g_m R_S} = \frac{1}{1/g_m + R_S}}$$

for $R_S >> 1/g_m$ ➡ $G_m \approx 1/R_S$

➡ **I_D is linearized at the cost of lower gain.**

Small Signal model including body effect and channel length modulation:



$$I_{out} = g_m V_1 - g_{mb} V_X - \frac{V_X}{r_O}$$

$$= g_m (V_{in} - I_{out} R_S) + g_{mb} (-I_{out} R_S) - \frac{I_{out} R_S}{r_O}$$

➡ $$\boxed{G_m = \frac{I_{out}}{V_{in}} = \frac{g_m r_O}{R_S + [1 + (g_m + g_{mb}) R_S] r_O}}$$

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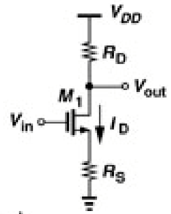
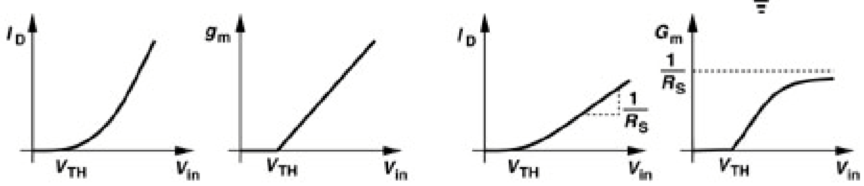
With and Without Source Degeneration

$$G_m = \frac{g_m r_O}{1 + [1 + (g_m + g_{mb}) R_S] r_O}$$



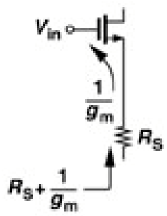


$R_S = 0$ $\qquad$ $R_S \neq 0$

Estimating Gain by Inspection

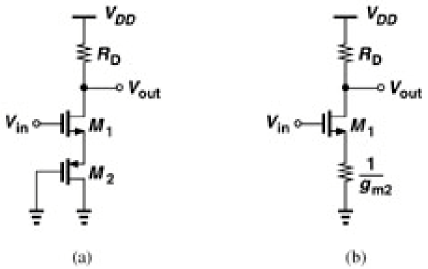
$$A_v = -\frac{g_m R_D}{1 + g_m R_S} = -\frac{R_D}{1/g_m + R_S}$$

$$Gain = -\frac{\text{Resistance seen at the Drain}}{\text{Total Resistance in the Source Path}}$$



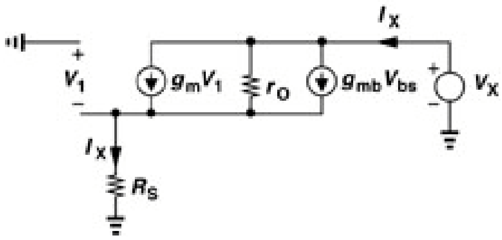
Example:

$$A_v = -\frac{R_D}{1/g_{m1} + 1/g_{m2}}$$



Output Resistance of Degenerated CS

$$V_1 = -I_X R_S$$



The current flowing in r_O:

$$I_X - (g_m + g_{mb})V_1$$
$$= I_X + (g_m + g_{mb})R_S I_X$$

➡ $$V_X = r_O[I_X + (g_m + g_{mb})R_S I_X] + I_X R_S$$

$$R_{out} = \frac{V_X}{I_X} = r_O[1 + (g_m + g_{mb})R_S] + R_S$$

$$R_{out} = [1 + (g_m + g_{mb})r_O]R_S + r_O$$

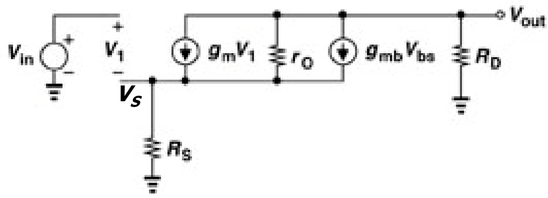
$$R_{out} \approx (g_m + g_{mb})r_O R_S + r_O$$ ➡ $$\boxed{R_{out} = [1 + (g_m + g_{mb})R_S]r_O}$$

Voltage Gain of Degenerated CS

The current through R_S must equal that through R_D:

$$I_{R_D} = I_{R_S} = \frac{V_{out}}{R_D}$$

➡ $$V_S = -V_{out}\frac{R_S}{R_D}$$



The current through r_O: $$I_{r_O} = -\frac{V_{out}}{R_D} - (g_m V_1 + g_{mb}V_{BS})$$

$$I_{r_O} = -\frac{V_{out}}{R_D} - [g_m(V_{in} + V_{out}\frac{R_S}{R_D}) + g_{mb}V_{out}\frac{R_S}{R_D}]$$ ➡ $$V_{out} = I_{r_O} r_O - \frac{V_{out}}{R_D}R_S$$

$$V_{out} = -\frac{V_{out}}{R_D}r_O - [g_m(V_{in} + V_{out}\frac{R_S}{R_D}) + g_{mb}V_{out}\frac{R_S}{R_D}]r_O - V_{out}\frac{R_S}{R_D}$$

$$\boxed{\frac{V_{out}}{V_{in}} = -\frac{g_m r_O R_D}{R_D + R_S + r_O + (g_m + g_{mb})R_S r_O}}$$

Voltage Gain of Degenerated CS

$$\frac{V_{out}}{V_{in}} = -\frac{g_m r_O R_D}{R_D + R_S + r_O + (g_m + g_{mb}) R_S r_O}$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_m r_O}{R_S + [1 + (g_m + g_{mb}) R_S] r_O} \frac{R_D [R_S + r_O + (g_m + g_{mb}) R_S r_O]}{R_D + R_S + r_O + (g_m + g_{mb}) R_S r_O}$$

$$\frac{V_{out}}{V_{in}} = G_m (R_{out} // R_D)$$

The output resistance of a degenerated CS stage: $R_{out} = [1 + (g_m + g_{mb}) R_S] r_O$

The Transconductance of a degenerated CS stage: $G_m = \frac{I_{out}}{V_{in}} = \frac{g_m r_O}{R_S + [1 + (g_m + g_{mb}) R_S] r_O}$

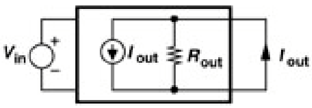
General expression to calculate A_v by inspection

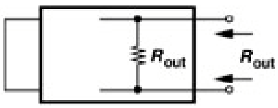
Lemma:

$$Av = -G_m R_{out}$$

G_m **: the transconductance of the circuit when the output is shorted to grounded.**

R_{out} **: the output resistance of the circuit when the input voltage is set to zero.**





- **For high voltage gain the output resistance must be high!**

  ➡ **A "buffer" is needed to drive a low-impedance load. The source follower can operate as a voltage buffer.**

Single Stage Amplifiers

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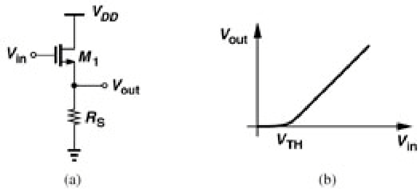
Source Follower (Common Drain)

Large Signal Behavior

M1 turns on in saturation:

$$V_{out} = I_D R_S$$

$$V_{out} = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{in} - V_{out} - V_{TH})^2 R_S$$



To calculate g_m :

$$\frac{\partial V_{out}}{\partial V_{in}} = \mu_n C_{ox}\frac{W}{L}(V_{in} - V_{out} - V_{TH})(1 - \frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH}}{\partial V_{in}})R_S$$

Since, $V_{TH} = V_{TH0} + \gamma\left(\sqrt{2\Phi_F + V_{SB}} - \sqrt{2\Phi_F}\right)$

$$\frac{\partial V_{TH}}{\partial V_{in}} = \frac{\partial V_{TH}}{\partial V_{SB}}\frac{\partial V_{SB}}{\partial V_{in}} = \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}\frac{\partial V_{SB}}{\partial V_{in}}$$

$$= \eta\frac{\partial V_{out}}{\partial V_{in}}$$

Source Follower Voltage Gain

$$\frac{\partial V_{out}}{\partial V_{in}} = \mu_n C_{ox} \frac{W}{L}(V_{in} - V_{out} - V_{TH})(1 - \frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH}}{\partial V_{in}})R_S$$

$$\frac{\partial V_{out}}{\partial V_{in}} = \mu_n C_{ox} \frac{W}{L}(V_{in} - V_{out} - V_{TH})(1 - \frac{\partial V_{out}}{\partial V_{in}} - \eta \frac{\partial V_{out}}{\partial V_{in}})R_S$$

$$\frac{\partial V_{out}}{\partial V_{in}} = \frac{\mu_n C_{ox} \frac{W}{L}(V_{in} - V_{out} - V_{TH})R_S}{1 + \mu_n C_{ox} \frac{W}{L}(V_{in} - V_{out} - V_{TH})R_S(1+\eta)}$$

We also have, $g_m = \mu_n C_{ox} \frac{W}{L}(V_{in} - V_{out} - V_{TH})$

⇨ $$Av = \frac{g_m R_S}{1 + (g_m + g_{mb})R_S}$$

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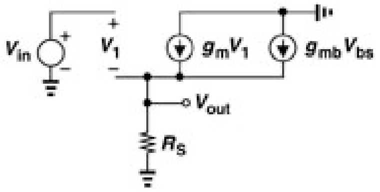
Source Follower Voltage Gain

Small Signal Equivalent Circuit

$$V_{out} = [g_m V_1 + g_{mb} V_{BS}]R_S$$
$$= [g_m (V_{in} - V_{out}) - g_{mb} V_{out}]R_S$$

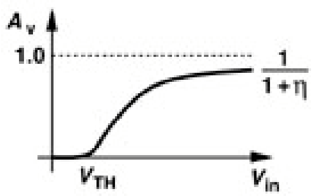


⇨ $$Av = \frac{V_{out}}{V_{in}} = \frac{g_m R_S}{1 + (g_m + g_{mb})R_S}$$

Since: $g_{mb} = \eta \, g_m$

And for : $g_m R_S >> 1$

⇨ $$Av \approx \frac{1}{(1+\eta)}$$



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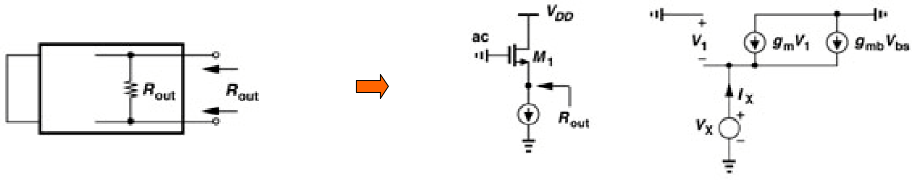
Source Follower Output Resistance

R_{out} **: the output resistance when the input voltage is set to zero.**



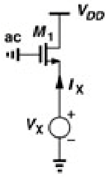
(a) (b)

$$V_1 = V_{BS} = -V_X$$

$$I_X - g_m V_X - g_{mb} V_X = 0$$

$$R_{out} = \frac{V_X}{I_X} = \frac{1}{g_m + g_{mb}}$$

Body Effect decreases the output resistance of source followers.



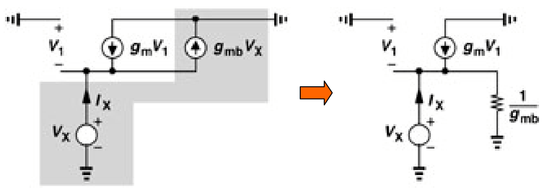
$$V_X \downarrow \;\Rightarrow\; V_{GS} \uparrow \;\; and \;\; V_{TH} \downarrow$$

$$\Rightarrow\; I_D \uparrow$$

Source Follower body effect

R_{out} **: the output resistance when the input voltage is set to zero.**

Small Signal Model Simplification

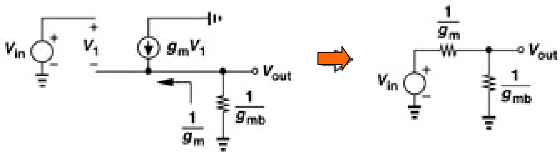


(a) (b)

Note that the value of the current source $g_{mb}V_{bs}$ is linearly proportional to the voltage across it.

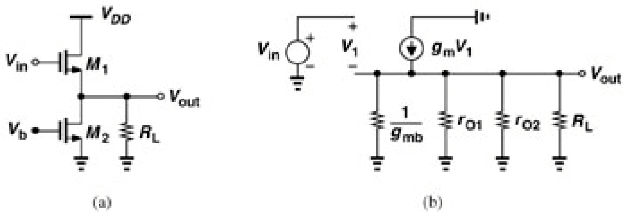
$$R_{out} = \frac{1}{g_m} // \frac{1}{g_{mb}} = \frac{1}{g_m + g_{mb}}$$

Source Follower Thévenin Equivalent



$$A_v = \frac{\frac{1}{g_{mb}}}{\frac{1}{g_m} + \frac{1}{g_{mb}}} = \frac{g_m}{g_m + g_{mb}}$$

Channel Length Modulation in M1 and M2



$$Av = \frac{\frac{1}{g_{mb}} // r_{O1} // r_{O2} // R_L}{\frac{1}{g_{mb}} // r_{O1} // r_{O2} // R_L + \frac{1}{g_m}}$$

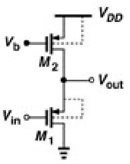
Source Follower Characteristics

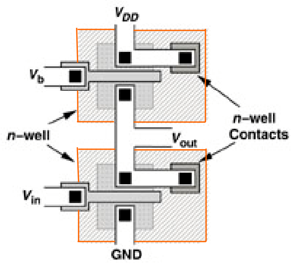
+ High input impedance and Moderate output impedance

- Nonlinearity

$$V_{TH} \; \alpha \sqrt{V_{SB}}$$

PMOS source follower with $V_{SB}=0$



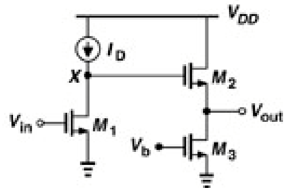


$$\mu_p < \mu_n \quad \Rightarrow \quad g_{mp} < g_{mn}$$

$$\Rightarrow \quad R_{outp} > R_{outn}$$

- Limited voltage swing

Example:



Without the source follower stage:

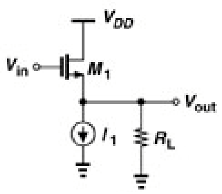
$$V_X > V_{GS1} - V_{TH1}$$

With the source follower stage:

$$V_X > V_{GS2} + (V_{GS3} - V_{TH3})$$

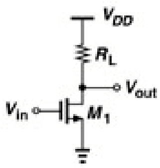
Low Load Impedance: CS vs SF

Source Follower Amplifier



$$A_{vSF} \approx \frac{R_L}{R_L + 1/g_m}$$

Common Source Amplifier



$$A_{vCS} \approx -g_m R_L$$

Assuming $R_L=1/g_m$

$$A_{vSF} \approx 1/2$$

$$A_{vCS} \approx -1$$

➡ **Source Followers are not necessarily efficient drivers.**

Single Stage Amplifiers

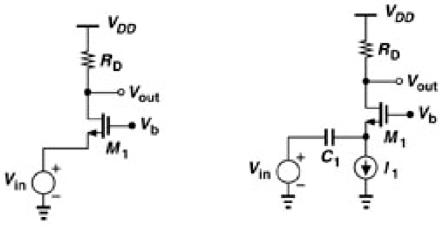
- **Basic Concepts**
- **Common Source Stage**
- **Source Follower**
- **Common Gate Stage**
- **Cascode Stage**

Hassan Aboushady
University of Paris VI

Common Gate Stage

Large Signal Behavior

$$V_{out} = V_{DD} - I_D R_D$$

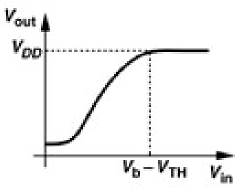


(a) (b)

Assuming M1 in saturation:

$$V_{out} = V_{DD} - \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_b - V_{in} - V_{TH})^2 R_D$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -\mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})\left(-1 - \frac{\partial V_{TH}}{\partial V_{in}}\right) R_D$$



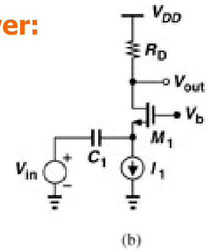
$$\frac{\partial V_{out}}{\partial V_{in}} = \mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})(1+\eta) R_D$$

$$\Rightarrow \boxed{A_v = g_m (1+\eta) R_D}$$

Common Gate Stage Input Resistance

Same as Output Resistance of Source Follower:

$$R_{in} = \frac{1}{g_m + g_{mb}}$$

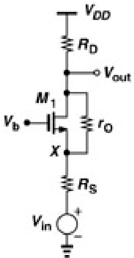


(b)

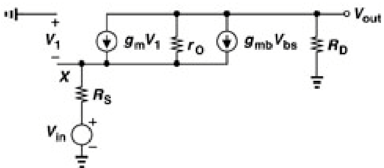
Body Effect:

- **increases A_v**
- **decreases R_{in}**

Common Gate Gain



Small Signal Signal Equivalent Circuit



The current through R_S is equal to $-V_{out}/R_D$: $V_1 - \frac{V_{out}}{R_D} R_S + V_{in} = 0$

The current through r_O is equal to $-V_{out}/R_D - g_m V_1 - g_{mb} V_1$:

$$r_O\left(\frac{-V_{out}}{R_D} - g_m V_1 - g_{mb} V_1\right) - \frac{V_{out}}{R_D} R_S + V_{in} = V_{out}$$

$$r_O\left[\frac{-V_{out}}{R_D} - (g_m + g_{mb})\left(V_{out}\frac{R_S}{R_D} - V_{in}\right)\right] - \frac{V_{out}}{R_D} R_S + V_{in} = V_{out}$$

Common Gate Gain

Common Gate Amplifier:

$$A_{vCG} = \frac{(g_m + g_{mb})r_O + 1}{R_D + R_S + r_O + (g_m + g_{mb})r_O R_S} R_D$$

Degenerated Common Source Amplifier:

$$A_{vCS} = -\frac{g_m r_O}{R_D + R_S + r_O + (g_m + g_{mb})r_O R_S} R_D$$

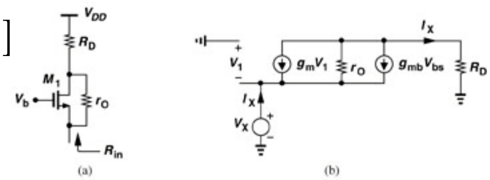
Common Gate Stage Input Resistance

Since $V_1 = -V_X$:

$$V_X = R_D I_X + r_O[I_X - (g_m + g_{mb})V_X]$$

$$\frac{V_X}{I_X} = \frac{R_D + r_O}{1 + (g_m + g_{mb})r_O}$$

$$R_{in} \approx \frac{R_D}{(g_m + g_{mb})r_O} + \frac{1}{(g_m + g_{mb})}$$

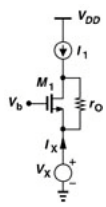


- **Assume $R_D = 0$:**

$$R_{in} = \frac{1}{1/r_O + (g_m + g_{mb})}$$

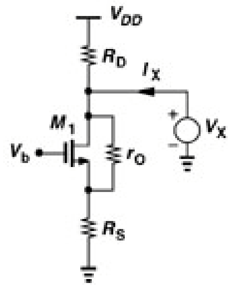
- **Replace R_D by ideal current source:**

$$R_{in} = \infty$$

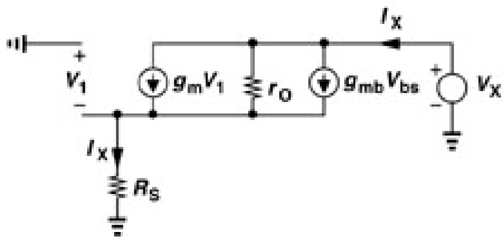


➡ **R_{in} of a common gate stage is low only if R_D is small.**

Common Gate Stage Output Impedance



Similar to Output Impedance of a Degenerated Common Source Stage



$$R_{out} = \left([1+(g_m+g_{mb})r_O]R_S + r_O\right) // R_D$$

Single Stage Amplifiers

- Basic Concepts
- Common Source Stage
- Source Follower
- Common Gate Stage
- Cascode Stage

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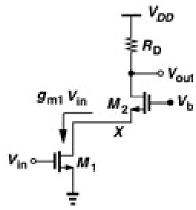
Biasing of a Cascode Stage

The cascade of CS stage and a CG stage is called "cascode".

M1 : the input device
M2 : the cascode device



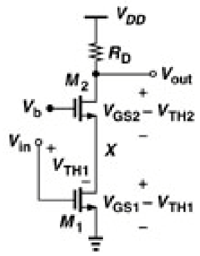
Biasing conditions:

- **M1 in saturation:**

$$V_X = V_b - V_{GS2}$$

$$V_b - V_{GS2} \geq V_{in} - V_{TH1}$$

$$V_b \geq V_{in} + V_{GS2} - V_{TH1}$$



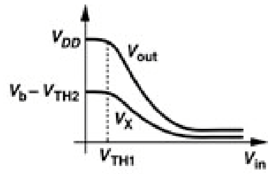
- **M2 in saturation:**

$$V_{out} - V_X \geq V_b - V_X - V_{TH2}$$

$$\boxed{V_{out} \geq V_{in} - V_{TH1} + V_{GS2} - V_{TH2}}$$

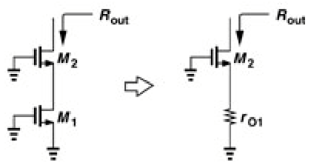
Cascode Stage Characteristics

Large signal behavior:
As V_{in} goes from zero to V_{DD}
For $V_{in} < V_{TH}$ M1 and M2 are OFF
⇨ $V_{out} = V_{DD}$



Output Resistance:

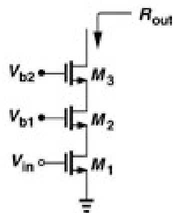
- **Same common source stage with a degeneration resistor equal to r_{O1}**



$$R_{out} = [1 + (g_{m2} + g_{mb2})r_{O2}]r_{O1} + r_{O2}$$

$$R_{out} \approx (g_{m2} + g_{mb2})r_{O2}r_{O1}$$

- **M2 boosts the output impedance of M1 by a factor of $g_m r_{O2}$**



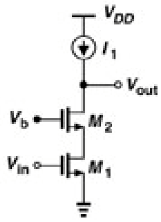
- **Triple cascode** R_{out} ↑↑

⇨ **difficult biasing at low supply voltage.**

Cascode Stage Voltage Gain

$$Av = -G_m R_{out}$$

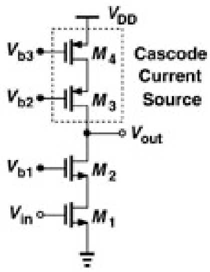
$$G_m \approx g_{m1}$$



Ideal Current Source:

$$R_{out} \approx (g_{m2} + g_{mb2}) r_{O2} r_{O1}$$

$$\boxed{A_v \approx (g_{m2} + g_{mb2}) r_{O2}\ g_{m1} r_{O1}}$$



Cascode Current Source:

$$R_{out} \approx g_{m2} r_{O2} r_{O1} \,//\, g_{m3} r_{O3} r_{O4}$$

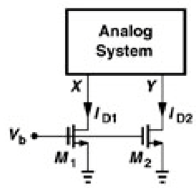
$$\boxed{A_v \approx g_{m1}\left(g_{m2} r_{O2} r_{O1} \,//\, g_{m3} r_{O3} r_{O4}\right)}$$

Shielding Property

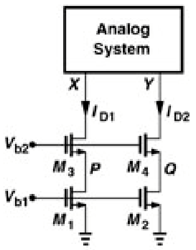
Assume V_X is higher than V_Y by ΔV.
Calculate the resulting difference between I_{D1} and I_{D2} (with $\lambda \neq 0$).

$$I_{D1} - I_{D2} = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_b - V_{TH})^2 (\lambda V_{DS1} - \lambda V_{DS2})$$

$$\boxed{I_{D1} - I_{D2} = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_b - V_{TH})^2 (\lambda\, \Delta V_{DS})}$$



(a)

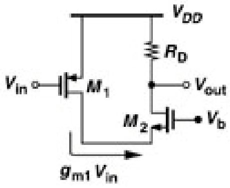


(b)

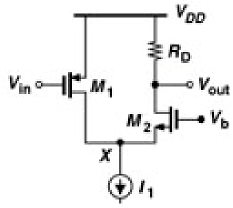
$$\Delta V_{PQ} = \Delta V \frac{r_{O1}}{[1 + (g_{m3} + g_{mb3}) r_{O3}] r_{O1} + r_{O3}} \approx \frac{\Delta V}{(g_{m3} + g_{mb3}) r_{O3}}$$

$$\boxed{I_{D1} - I_{D2} = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_b - V_{TH})^2 \frac{\lambda\, \Delta V}{(g_{m3} + g_{mb3}) r_{O3}}}$$

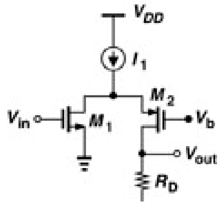
Folded Cascode



Simple Folded Cascode

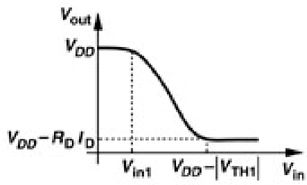


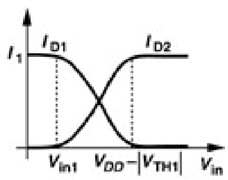
Folded Cascode with biasing



Folded Cascode with NMOS input

Large Signal Characteristics:

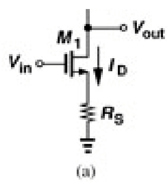




Output Resistance of Folded Cascode

Degenerated Common Source Stage:

$$R_{out} = [1+(g_{m1}+g_{mb1})r_{O1}]R_S + r_{O1}$$



Folded Cascode Stage:

M_1 → M_2

R_S → $r_{O1} // r_{O3}$

$$R_{out} = [1+(g_{m2}+g_{mb2})r_{O2}](r_{O1} // r_{O3}) + r_{O2}$$