Differential Amplifiers

- ***Single Ended and Differential Operation***
- ***Basic Differential Pair***
- ***Common-Mode Response***
- ***Differential Pair with MOS loads***

Hassan Aboushady
University of Paris VI

References

- **B. Razavi, "Design of Analog CMOS Integrated Circuits", McGraw-Hill, 2001.**

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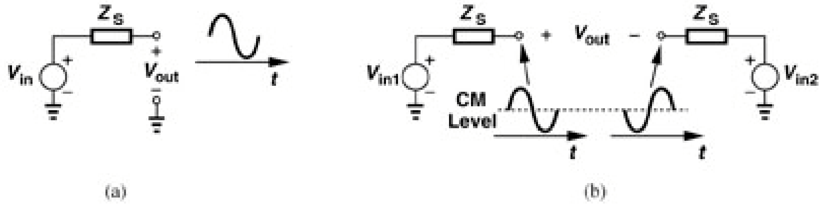
Single Ended and Differential Operation

- **Single Ended Signal:**

- Measured with respect to a fixed potential, usually ground.

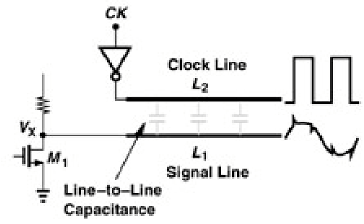
- **Differential Signal:**

- Measured between 2 nodes that have equal and opposite excursions around a fixed potential.
- The center potential is called "Common Mode" (CM).



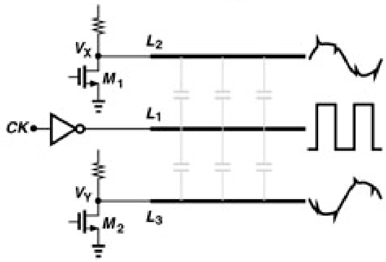
Rejection of Common Mode Noise

- **Single Ended Signal:**
  - Due to capacitive coupling, transitions on the clock line corrupt the signal on L1 .



(a)

- **Differential Signal:**
  - If the clock line is placed midway, the transitions disturb the differential signals by equal amounts, leaving the difference intact.



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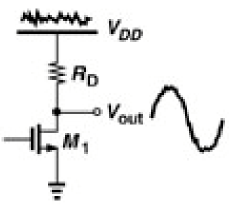
Rejection of Power Supply Noise



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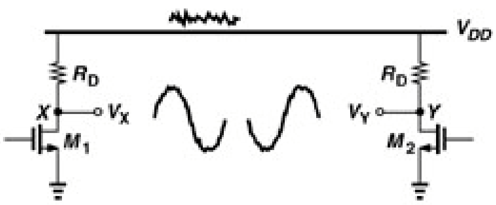


(b)

- **Maximum Output Swing:**

$$V_{out\,\max} = V_{DD} - (V_{GS} - V_{TH})$$

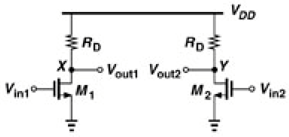
- **Maximum Output Swing:**

$$V_{X\max} - V_{Y\max} = 2[V_{DD} - (V_{GS} - V_{TH})]$$

Differential Pair

Differential circuit ➡ **sensitive to the input CM level.**

$$if \quad V_{in1} \neq V_{in2}$$
$$\Rightarrow \quad I_{D1} \neq I_{D2}$$
$$\Rightarrow \quad g_{m1} \neq g_{m2}$$



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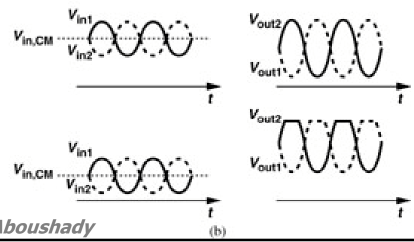
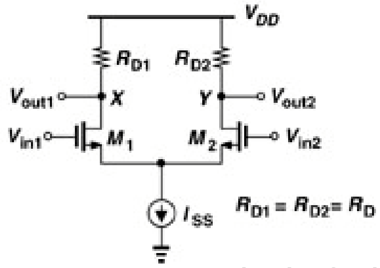


(b)

Differential pair ➡ **minimal dependence on input CM level.**

$$I_{SS} = I_{D1} + I_{D2}$$
$$if \quad V_{in1} = V_{in2}$$
$$\Rightarrow \quad I_{D1} = I_{D2} = \frac{I_{SS}}{2}$$
$$Output\ CM = V_{DD} - R_D \frac{I_{SS}}{2}$$



Differential Pair: Qualitative Analysis

$V_{in1} << V_{in2}$ ➡ **M1 OFF, M2 ON**

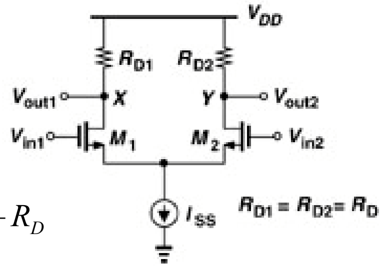
$I_{D2} = I_{SS}$ ➡ $V_{out1} = V_{DD}$, $V_{out2} = V_{DD} - I_{SS} R_D$

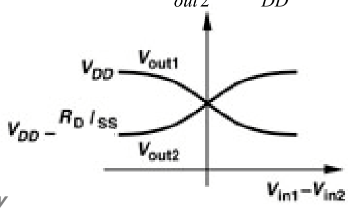
$V_{in1} = V_{in2}$ ➡ **M1 ON, M2 ON**

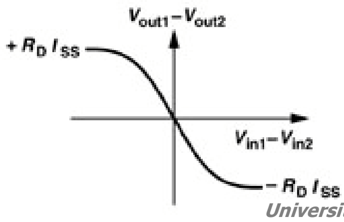
$I_{D1} = I_{D2} = \frac{I_{SS}}{2}$ ➡ $V_{out1} = V_{out2} = V_{DD} - \frac{I_{SS}}{2} R_D$

$V_{in1} >> V_{in2}$ ➡ **M1 ON, M2 OFF**

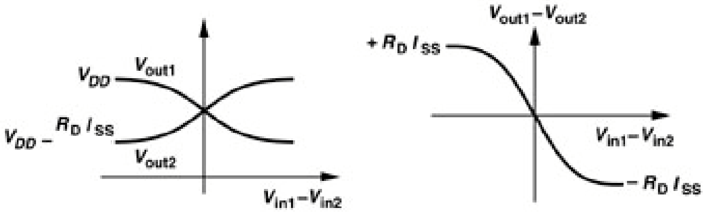
$I_{D1} = I_{SS}$ ➡ $V_{out1} = V_{DD} - I_{SS} R_D$, $V_{out2} = V_{DD}$







Differential Pair: Qualitative Analysis



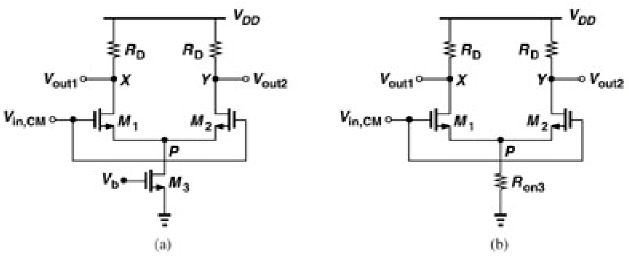
- Maximum and minimum levels are well-defined and independent of the input CM: V_{DD} and V_{DD} - $R_D I_{SS}$
- The small signal gain (the slope of V_{out1}-V_{out2} vs V_{in1}-V_{in2}) is maximum for V_{in1}=V_{in2} (equilibrium).

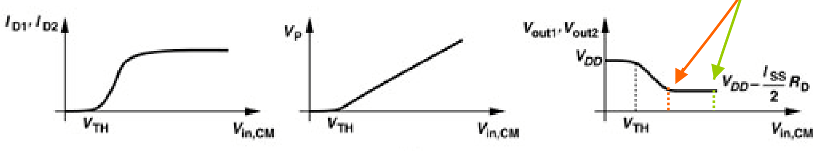
Differential Pair: Common-Mode Behavior

To study Common-Mode ⇨ $V_{in1} = V_{in2} = V_{in,CM}$

For proper operation:

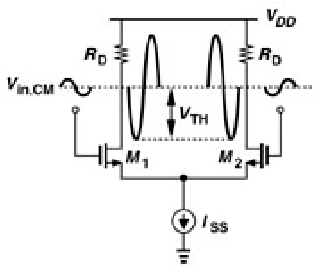
- M3 in saturation ⇨ $V_{in,CM} \geq V_{GS1} + (V_{GS3} - V_{TH3})$
- M1 & M2 in saturation ⇨ $V_{in,CM} \leq V_{DD} - R_D \frac{I_{SS}}{2} + V_{TH}$





(c)

Differential Pair: Output Voltage Swing



For M1 & M2 in saturation:

$$V_{out} - V_P \geq V_{in,CM} - V_P - V_{TH}$$

$$V_{out} \geq V_{in,CM} - V_{TH}$$

Output Voltage Swing:

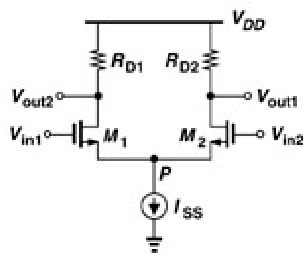
$$\boxed{V_{DD} \geq V_{out} \geq V_{in,CM} - V_{TH}}$$

➡ **To increase output swing, we choose a low** $V_{in,CM}$

Differential Pair: Quantitative Analysis

$$V_P = V_{in1} - V_{GS1} = V_{in2} - V_{GS2}$$

$$\boxed{V_{in1} - V_{in2} = V_{GS1} - V_{GS2}} \quad (1)$$



Assuming M1 & M2 in saturation:

$$(V_{GS} - V_{TH})^2 = \frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}$$

$$\boxed{V_{GS} = \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}} + V_{TH}} \quad (2)$$

From eq. 1 & 2 :

➡ $$V_{in1} - V_{in2} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} \frac{W}{L}}}$$

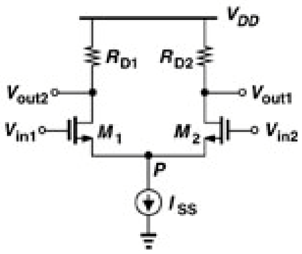
Squaring the 2 sides, and since: $I_{SS} = I_{D1} + I_{D2}$

$$\boxed{(V_{in1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox} \frac{W}{L}} (I_{SS} - 2\sqrt{I_{D1} I_{D2}})}$$

Differential Pair: Quantitative Analysis

Previous equation can be written:

$$\frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{in1}-V_{in2})^2 - I_{SS} = -2\sqrt{I_{D1}I_{D2}}$$



Squaring the 2 sides, and since:

$$4I_{D1}I_{D2} = (I_{D1}+I_{D2})^2 - (I_{D1}-I_{D2})^2$$
$$= I_{SS}^2 - (I_{D1}-I_{D2})^2$$

We arrive at:

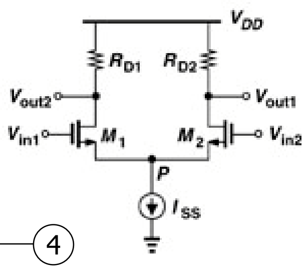
$$(I_{D1}-I_{D2})^2 = -\frac{1}{4}\left(\mu_n C_{ox}\frac{W}{L}\right)^2 (V_{in1}-V_{in2})^4 + I_{SS}\mu_n C_{ox}\frac{W}{L}(V_{in1}-V_{in2})^2$$

$$I_{D1}-I_{D2} = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{in1}-V_{in2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(V_{in1}-V_{in2})^2} \qquad (3)$$

Differential Pair: Quantitative Analysis

Let $\Delta V_{in} = V_{in1} - V_{in2}$ and $\Delta I_D = I_{D1} - I_{D2}$

Deriving eq. 3 with respect to ΔV_{in}



$$G_m = \frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{\mu_n C_{ox}}{2}\frac{W}{L}\frac{\frac{4I_{SS}}{\mu_n C_{ox}W/L}-2\Delta V_{in}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}W/L}-\Delta V_{in}^2}} \qquad (4)$$

For $\Delta V_{in} = 0, \quad G_m = \sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}$

Since: $V_{out1} - V_{out2} = V_{DD} - I_{D1}R_{D1} - V_{DD} - I_{D2}R_{D2}$

$$\Delta V_{out} = \Delta I_D R_D \quad \Rightarrow \quad \Delta V_{out} = G_m \Delta V_{in} R_D$$

The small signal differential voltage gain:

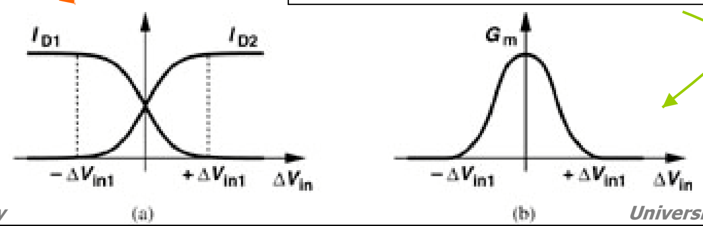
$$|A_v| = \frac{\Delta V_{out}}{\Delta V_{in}} = G_m R_D = \sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}R_D$$

Drain Currents and Overall Transconductance

ΔV_{in1} is when $I_{D1} = I_{SS}$ ➡ $\Delta V_{in1} = V_{GS1} - V_{TH1}$ ➡ $\Delta V_{in1} = \sqrt{\dfrac{2I_{SS}}{\mu_n C_{ox}\dfrac{W}{L}}}$

$$I_{D1} - I_{D2} = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{in1} - V_{in2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (V_{in1} - V_{in2})^2} \quad (3)$$

$$G_m = \frac{\mu_n C_{ox}}{2}\frac{W}{L}\frac{\dfrac{4I_{SS}}{\mu_n C_{ox} W/L} - 2\Delta V_{in}^2}{\sqrt{\dfrac{4I_{SS}}{\mu_n C_{ox} W/L} - \Delta V_{in}^2}} \quad (4)$$



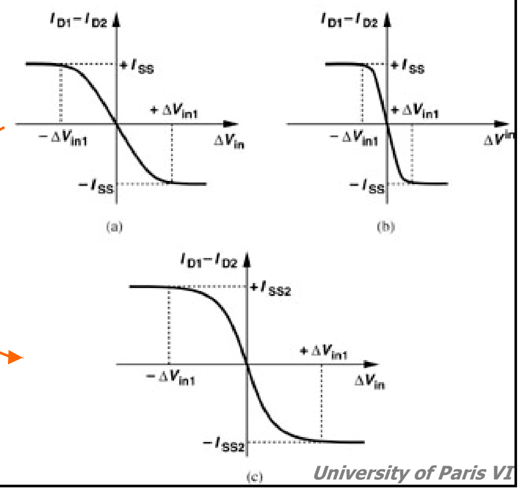
ΔI_D vs ΔV_D

Plot the input-output characteristics of a differential pair as the device width and the tail current vary:

$$\Delta V_{in1} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$$

$\dfrac{W}{L}$ ↑↑

I_{SS} ↑↑



Differential Pair: Small Signal Gain

$$|A_v| = \frac{\Delta V_{out}}{\Delta V_{in}} = G_m R_D = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} R_D$$

In equilibrium, we have

$$I_{D1} = I_{D2} = \frac{I_{SS}}{2}$$

$A_v = g_m R_D$ **Where gm is the transconductance of M1 & M2.**

Calculating Small Signal Gain by Superposition

Set V_{in2} =0
M1 forms a common source stage with a degeneration resistance

$$A_v = \frac{V_X}{V_{in1}} = -\frac{g_{m1} R_D}{1 + g_{m1} R_S}$$

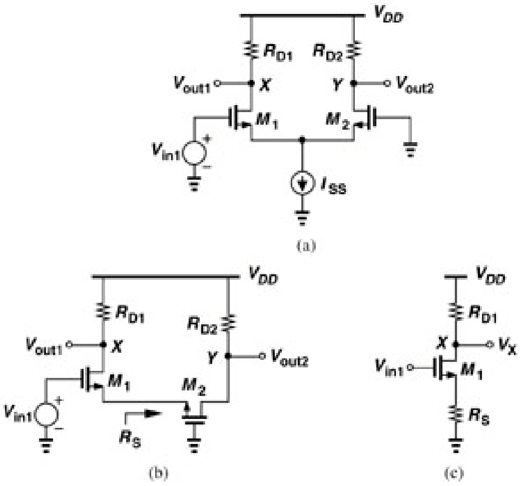
Neglecting channel length modulation and body effect

$$R_S = 1/g_{m2}$$

$$\frac{V_X}{V_{in1}} = -\frac{g_{m1} R_D}{1 + g_{m1}/g_{m2}}$$

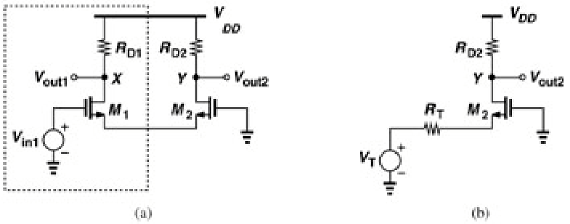
$$\Rightarrow \frac{V_X}{V_{in1}} = -\frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

5



Calculating Small Signal Gain by Superposition

Replacing M1 by its Thévenin equivalent:



$V_T = V_{in1}$ $R_T = 1/g_{m1}$ $R_{in2} = 1/g_{m2}$

$$\frac{V_Y}{V_{in1}} = \frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} \quad (6)$$

From eq. 5 & 6, we get:

$$(V_X - V_Y)\Big|_{due\ to\ V_{in1}} = \frac{-2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} V_{in1}$$

Calculating Small Signal Gain by Superposition

$$(V_X - V_Y)\Big|_{due\ to\ V_{in1}} = \frac{-2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} V_{in1}$$

Since: $g_{m1} = g_{m2} = g_m$

$$(V_X - V_Y)\Big|_{due\ to\ V_{in1}} = -g_m R_D V_{in1}$$

Similarly we can say that:

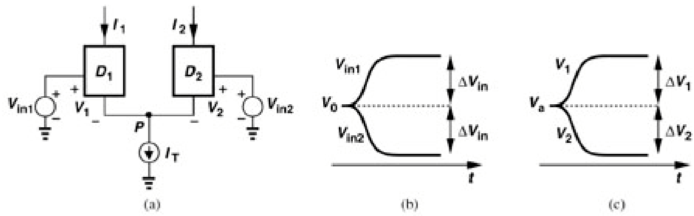
$$(V_X - V_Y)\Big|_{due\ to\ V_{in2}} = g_m R_D V_{in2}$$

The small signal differential voltage gain:

$$\frac{(V_X - V_Y)\Big|_{total}}{V_{in1} - V_{in2}} = -g_m R_D$$

The concept of Half Circuit

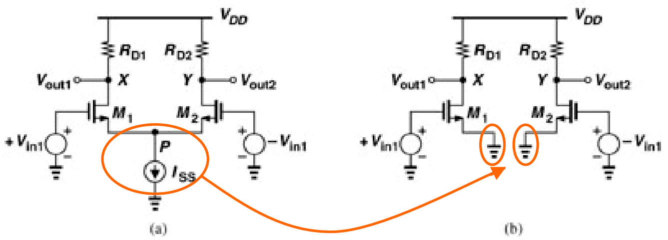
If a fully symmetric differential pair senses differential inputs then the concept of half circuit can be applied.



- ***A differential change in the inputs V_{in1} and V_{in2} is absorbed by V_1 and V_2 leaving V_P constant***

Application of The Half Circuit Concept

Since V_P experiences no change, node P can be considered "ac ground" and the circuit can be decomposed into two separate halves



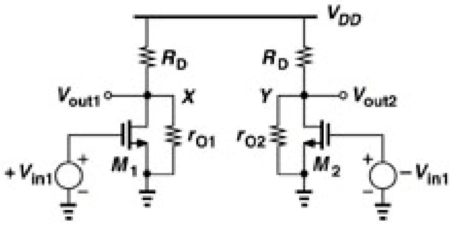
➡ **Two common source amplifiers:**

$$\frac{V_X}{V_{in1}} = -g_m R_D \qquad \frac{V_Y}{V_{in2}} = -g_m R_D$$

$$\boxed{\frac{V_X - V_Y}{V_{in1} - V_{in2}} = -g_m R_D}$$

The Half Circuit Concept : Example

Taking into account the output resistance (channel length modulation)



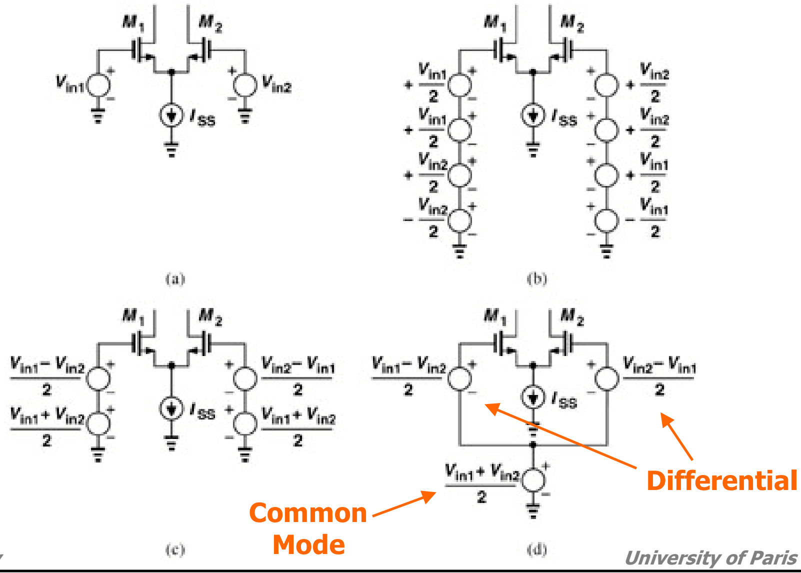
➡ **Two common source amplifiers:**

$$\frac{V_X}{V_{in1}} = -g_m\left(R_D // r_{O1}\right) \qquad \frac{V_Y}{V_{in2}} = -g_m\left(R_D // r_{O2}\right)$$

$$\boxed{\frac{V_X - V_Y}{V_{in1} - V_{in2}} = -g_m\left(R_D // r_O\right)}$$

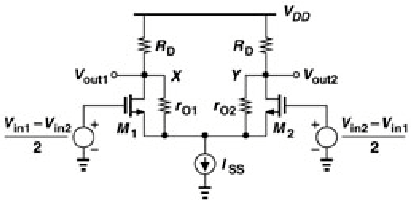
Arbitrary Inputs to a Differential Pair

Conversion of arbitrary inputs to differential and common-mode components:



Arbitrary Inputs to a Differential Pair: Example

Calculate V_X and V_Y if $V_{in1} \neq V_{in2}$ and $\lambda \neq 0$

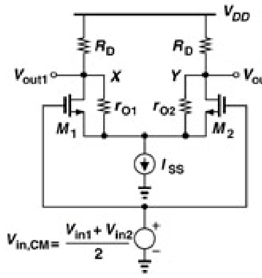


For differential mode operation:

$$V_X = -g_m (R_D // r_{O1}) \frac{(V_{in1} - V_{in2})}{2}$$

$$V_Y = -g_m (R_D // r_{O2}) \frac{(V_{in2} - V_{in1})}{2}$$

$$\boxed{V_X - V_Y = -g_m (R_D // r_O)(V_{in1} - V_{in2})}$$



For common mode operation:

$$I_{D1} = I_{D2} = I_{SS}/2$$

Assuming fully symmetric circuit and Ideal Current Source:

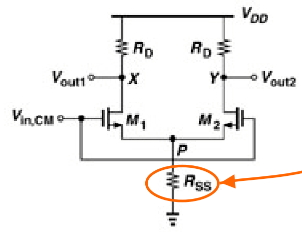
- I_{D1} **and** I_{D2} **independent of** $V_{CM,in}$
- V_X **and** V_Y **independent of** $V_{CM,in}$

The Differential pair circuit:

Amplifies $V_{in1} - V_{in2}$ **, Eliminates the effect of** $V_{CM,in}$

Common Mode Response: Non-Ideal Current Source

Assuming fully symmetric circuit with finite output impedance current source, R_{SS} :

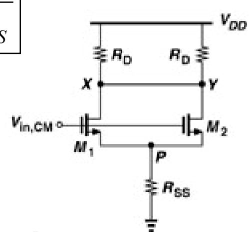


(a)

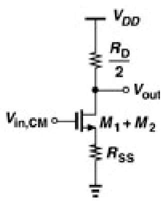
Equivalent circuit:

- **Degenerated Common Source**

$$\boxed{A_{v,CM} = \frac{V_{out}}{V_{in,CM}} = -\frac{R_D/2}{1/(2g_m) + R_{SS}}}$$



(b)



(c)

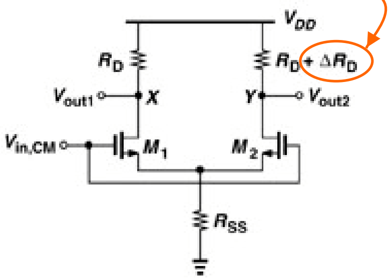
g_m the transconductance of 1 transistor.

Common Mode Response: R_D Mismatch Effect

Assuming M1 & M2 identical:

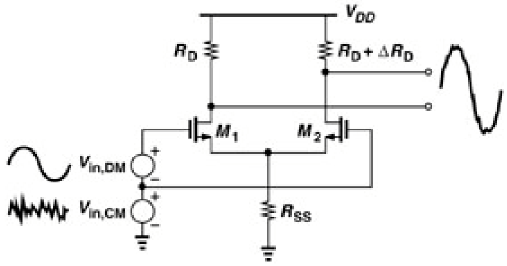
$$\Delta V_X = -\Delta V_{in,CM}\frac{R_D}{1/(2g_m)+R_{SS}}$$

$$\Delta V_Y = -\Delta V_{in,CM}\frac{R_D+\Delta R_D}{1/(2g_m)+R_{SS}}$$



➡ **Common mode to differential conversion**

Effect of CM noise in the presence of resistor noise



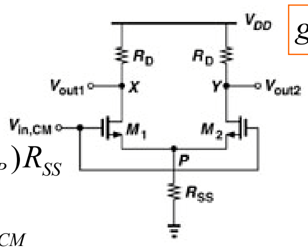
Common Mode Response: M_1-M_2 Mismatch Effect

$$I_{D1} = g_{m1}(V_{in,CM}-V_P)$$

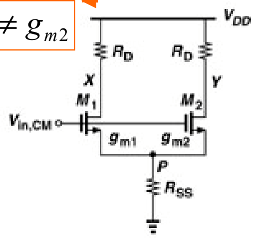
$$I_{D2} = g_{m2}(V_{in,CM}-V_P)$$

$$V_P = (g_{m2}+g_{m2})(V_{in,CM}-V_P)R_{SS}$$

$$V_P = \frac{(g_{m2}+g_{m2})R_{SS}}{(g_{m2}+g_{m2})R_{SS}+1}V_{in,CM}$$



$g_{m1} \neq g_{m2}$



$$V_X = -g_{m1}(V_{in,CM}-V_P)R_D \quad \Rightarrow \quad V_X = \frac{-g_{m1}}{(g_{m1}+g_{m2})R_{SS}+1}R_D V_{in,CM}$$

$$V_Y = -g_{m2}(V_{in,CM}-V_P)R_D \quad \Rightarrow \quad V_Y = \frac{-g_{m2}}{(g_{m1}+g_{m2})R_{SS}+1}R_D V_{in,CM}$$

$$V_X - V_Y = \frac{(g_{m1}-g_{m2})R_D V_{in,CM}}{(g_{m1}+g_{m2})R_{SS}+1}$$

CM to DM Conversion Gain: ➡ $A_{CM-DM} = \dfrac{V_X - V_Y}{V_{in,CM}} = \dfrac{\Delta g_m R_D}{(g_{m1}+g_{m2})R_{SS}+1}$

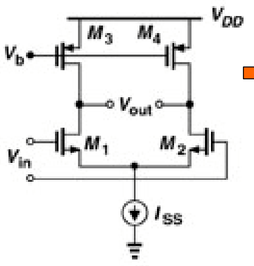
Common Mode Rejection Ratio

$$CMRR = \left| \frac{A_{DM}}{A_{CM-DM}} \right|$$

A_{DM} : **Differential Mode Gain**

A_{CM-DM} : **Common-Mode to Differential Mode Gain**

Cascode Differential Pair



(b)

Current Source Load:

$$A_v = g_{mN} (r_{ON} // r_{OP})$$

Low gain 10 to 20.

To increase the gain:
Cascode Differential Pair

$$A_v = g_{m1} (g_{m3} r_{O3} r_{O1} // g_{m5} r_{O5} r_{O7})$$