Frequency Response of Amplifiers

- ***General Considerations***
  - ***Miller Effect***
  - ***Association of Poles with Nodes***
- ***Common Source Stage***
- ***Source Follower***
- ***Differential Pair***

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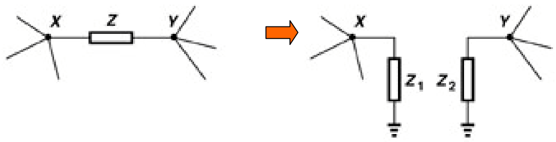
References

- **B. Razavi, "Design of Analog CMOS Integrated Circuits", McGraw-Hill, 2001.**

Miller Effect

- **Miller's Theorem**



(a) (b)

with $A_v = \dfrac{V_Y}{V_X}$ **we have** $\boxed{Z_1 = \dfrac{Z}{1 - A_v}}$ $\boxed{Z_2 = \dfrac{Z}{1 - A_v^{-1}}}$

- **Proof**

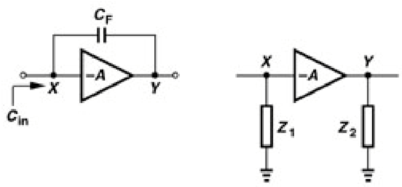
$$\frac{V_X - V_Y}{Z} = \frac{V_X}{Z_1} \quad \Rightarrow \quad Z_1 = \frac{Z}{1 - \dfrac{V_Y}{V_X}}$$

$$\frac{V_Y - V_X}{Z} = \frac{V_Y}{Z_2} \quad \Rightarrow \quad Z_2 = \frac{Z}{1 - \dfrac{V_X}{V_Y}}$$

Example 1

- **Calculate the input capacitance Cin:**

$$Z = \frac{1}{sC_F} \qquad Z_1 = \frac{\dfrac{1}{sC_F}}{1 + A}$$



(a) (b)

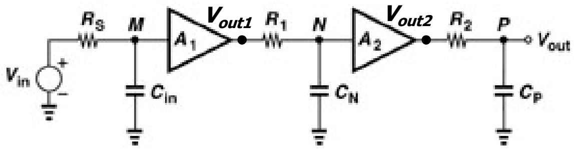
$$\Rightarrow \quad C_{in} = C_F(1 + A)$$

$Av = \dfrac{V_Y}{V_X}$ **should be calculated at the frequency of interest.**

To simplify calculations we usually use low frequency value of Av.

Miller's theorem cannot be used simultaneously to calculate input-output transfer function and the output impedance.

Association of Poles with Nodes



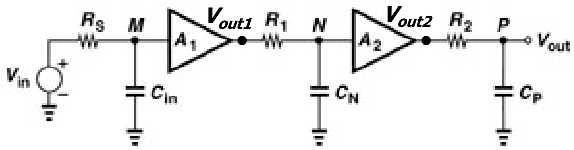
$$V_M(s) = \frac{V_{in}(s)}{R_S + \frac{1}{sC_{in}}} \frac{1}{sC_{in}} = \frac{V_{in}(s)}{1 + sR_S C_{in}}$$

$$V_N(s) = \frac{V_{out1}(s)}{1 + sR_1 C_N}$$

$$V_P(s) = \frac{V_{out2}(s)}{1 + sR_2 C_P}$$

$$\Rightarrow \quad \boxed{\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1 + sR_S C_{in}} \frac{A_2}{1 + sR_1 C_N} \frac{1}{1 + sR_2 C_P}}$$

Association of Poles with Nodes



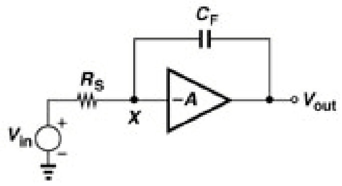
$$\boxed{\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1 + sR_S C_{in}} \frac{A_2}{1 + sR_1 C_N} \frac{1}{1 + sR_2 C_P}}$$

$$\omega_1 = \frac{1}{R_S C_{in}} \qquad \omega_2 = \frac{1}{R_1 C_N} \qquad \omega_3 = \frac{1}{R_2 C_P}$$

3 poles:
each determined by the total capacitance seen from each node to ground multiplied by the total resistance seen at the node to ground

Example 2

- **Calculate the pole associated with node X:**



The total equivalent capacitance seen from X to ground: $C_X = C_F(1+A)$

The pole frequency:

$$\omega_X = \frac{1}{R_S C_X} = \frac{1}{R_S C_F(1+A)}$$

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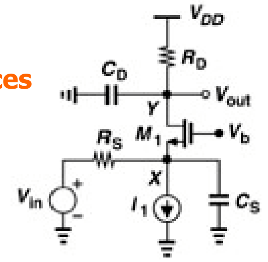
Example 3

- **Neglecting channel length modulation, compute the transfer function of the common gate stage with parasitic capacitances**



Parasitic capacitances at node X: $C_S = C_{GS1} + C_{SB1}$

Input resistance of a common gate amplifier: $R_{in} = \dfrac{1}{g_{m1} + g_{mb1}}$

Pole frequency at node X:

$$\omega_X = \frac{1}{\left(C_{GS1} + C_{SB1}\right)\left(R_S \,//\, \dfrac{1}{g_{m1} + g_{mb1}}\right)}$$

Parasitic capacitances at node Y: $C_D = C_{DG1} + C_{DB1}$

Pole frequency at node X:

$$\omega_Y = \frac{1}{(C_{DG1} + C_{DB1})R_D}$$

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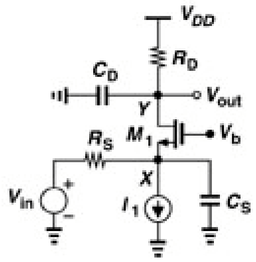
Example 3 (cont.)

Low-frequency gain of a common gate stage neglecting channel length modulation:

$$A_{v0} = \frac{(g_m + g_{mb})R_D}{1 + (g_{m1} + g_{mb1})R_S}$$



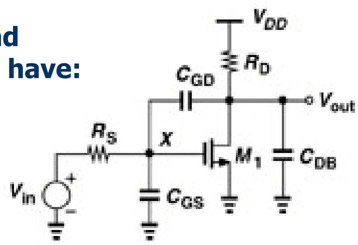
The overall transfer function is given by:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{A_{v0}}{\left(1 + \frac{s}{\omega_{in}}\right)\left(1 + \frac{s}{\omega_{out}}\right)}$$

Note that if we do not neglect r_{01}, the input and output nodes interact, making it difficult to calculate the poles.

Common Source Stage

Neglecting channel length modulation and applying the Miller's theorem on C_{GD}, we have:



The total capacitance at node X:

$$C_X = C_{GS} + (1 - A_v)C_{GD}$$

where, $A_v = -g_m R_D$

The 1st pole frequency:

$$\omega_{p1} = \frac{1}{R_S\left(C_{GS} + (1 + g_m R_D)C_{GD}\right)}$$

The total capacitance at the output node:

$$C_{out} = C_{DB} + \left(1 - A_v^{-1}\right)C_{GD} \approx C_{DB} + C_{GD}$$

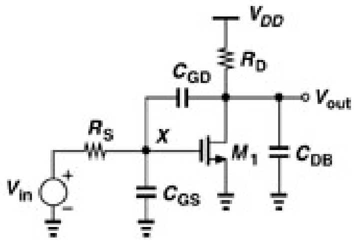
The 2nd pole frequency:

$$\omega_{p2} = \frac{1}{R_D\left(C_{DB} + C_{GD}\right)}$$

Common Source Stage

The transfer function:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{-g_m R_D}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$



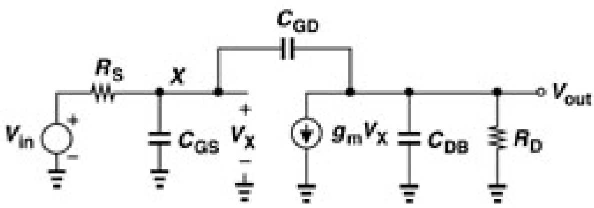
r_0 and any load capacitance can be easily included.

Sources of error (approximation):

- **we have not considered the existence of zeros in the circuit**
- **the amplifier gain varies with frequency**

Common Source : "exact " Transfer Function

To obtain the exact transfer function:



Applying Kirchoff Current Law (KCL):

$$\frac{V_X - V_{in}}{R_S} + sC_{GS}V_X + sC_{GD}(V_X - V_{out}) = 0$$

$$sC_{GD}(V_{out} - V_X) + g_m V_X + \left(sC_{DB} + \frac{1}{R_D}\right)V_{out} = 0$$

Common Source : "exact" 1st pole

After some manipulations, we get:

$$\frac{V_{out}}{V_{in}} = \frac{(sC_{GD} - g_m)R_D}{R_S R_D \xi\, s^2 + [R_S(1+g_m R_D)C_{GD} + R_S C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})]s + 1}$$

with $\xi = C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB}$

Writing the denominator as: $D = \left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p1}}\right) = \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1$

Assuming $|\omega_{p1}| << |\omega_{p2}|$

$$\Rightarrow \quad \omega_{p1} \approx \frac{1}{R_S(1+g_m R_D)C_{GD} + R_S C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})}$$

Compare this result with ω_{in} calculated using Miller's Theorem

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Common Source : "exact" 2nd pole

$$\frac{V_{out}}{V_{in}} = \frac{(sC_{GD} - g_m)R_D}{R_S R_D \xi\, s^2 + [R_S(1+g_m R_D)C_{GD} + R_S C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})]s + 1}$$

with $\xi = C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB}$

having $D = \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1$

and $\omega_{p1} \approx \frac{1}{R_S(1+g_m R_D)C_{GD} + R_S C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})}$

then $\omega_{p2} = \frac{1}{R_S R_D \xi}\frac{1}{\omega_{p1}}$

$$\Rightarrow \quad \omega_{p2} = \frac{R_S(1+g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})}{R_S R_D(C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB})}$$

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Comparison between "exact" and Miller's theorem

1st pole:

If $R_D(C_{GD}+C_{DB})$ **is negligible**

exact

$$\omega_{p1}=\frac{1}{R_S\left(C_{GS}+(1+g_mR_D)C_{GD}\right)+R_D\left(C_{GD}+C_{DB}\right)}$$

Miller

$$\omega_{p1}=\frac{1}{R_S\left(C_{GS}+(1+g_mR_D)C_{GD}\right)}$$

2nd pole:

exact

$$\omega_{p2}=\frac{R_S(1+g_mR_D)C_{GD}+R_SC_{GS}+R_D(C_{GD}+C_{DB})}{R_SR_D\left(C_{GS}C_{GD}+C_{GS}C_{DB}+C_{GD}C_{DB}\right)}$$

$$if\ C_{GS}>>(1+g_mR_D)C_{GD}+\frac{R_D}{R_S}(C_{GD}+C_{DB})$$

$$\omega_{p2}\approx\frac{C_{GS}}{R_D\left(C_{GS}C_{GD}+C_{GS}C_{DB}+C_{GD}C_{DB}\right)}$$

Miller

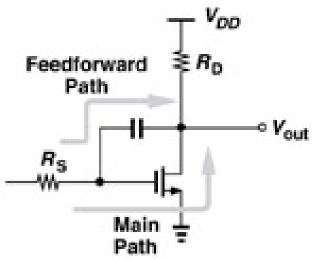
$$\omega_{p2}=\frac{1}{R_D\left(C_{DB}+C_{GD}\right)}$$

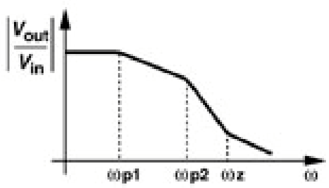
Common Source : transfer function zero

After some manipulations, we get:

$$\frac{V_{out}}{V_{in}}=\frac{(sC_{GD}-g_m)R_D}{R_SR_D\xi\, s^2+\left[R_S(1+g_mR_D)C_{GD}+R_SC_{GD}+R_SC_{GS}+R_D(C_{GD}+C_{DB})\right]s+1}$$

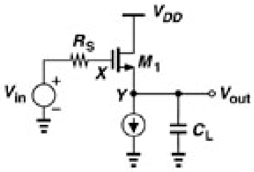
$$\omega_z=\frac{g_m}{C_{GD}}$$

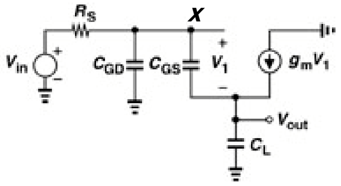




Source Follower

High frequency equivalent circuit





Applying Kirchoff Current Law (KCL)

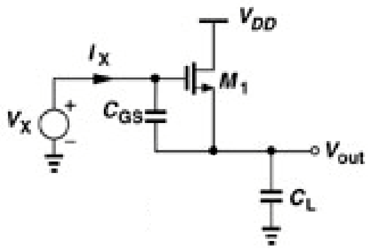
at the output node: $sC_{GS}V_1 + g_m V_1 = sC_L V_{out}$

at node X: $\dfrac{V_{out} + V_1 - V_{in}}{R_S} + sC_{GD}(V_{out} + V_1) + sC_{GS}V_1 = 0$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{g_m + sC_{GS}}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_S C_{GD} + C_L + C_{GS})s + g_m}$$

Source Follower Input Impedance

$$V_X = \frac{I_X}{sC_{GS}} + \left(I_X + \frac{g_m I_X}{sC_{GS}}\right)\frac{1}{sC_L}$$



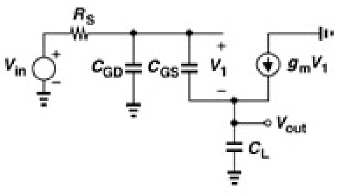
Input Impedance:

$$Z_{in} = \frac{1}{sC_{GS}} + \frac{1}{sC_L} + \frac{g_m}{s^2 C_{GS} C_L}$$

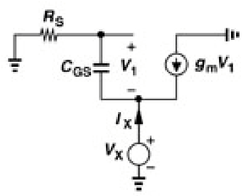
Note the negative resistance: $\dfrac{-g_m}{\omega^2 C_{GS} C_L}$

Source Follower Output Impedance

Neglecting C_{GD}



(b)

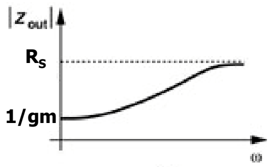


(a)

Applying KCL

$$sC_{GS}V_1 + g_mV_1 + I_X = 0$$

$$\frac{V_1}{R_S} + sC_{GS}(V_1 - V_X) = 0$$



(c)

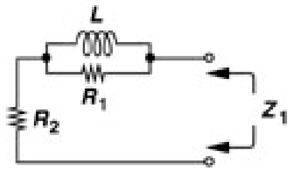
Output Impedance:

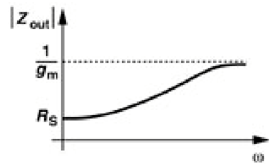
$$Z_{out} = \frac{sC_{GS}R_S + 1}{sC_{GS} + g_m}$$

Z_{out} increases with frequency

It contains an inductive component

Source Follower Output Impedance Equivalent C^t





Equivalent circuit of source follower output impedance:

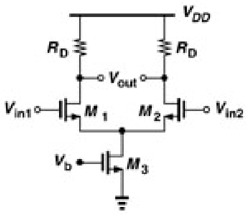
$$at \quad \omega = \infty \quad \Rightarrow \quad Z_1 = R_1 + R_2$$

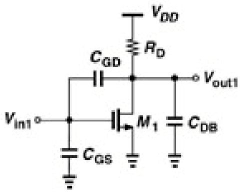
$$at \quad \omega = 0 \quad \Rightarrow \quad Z_1 = R_2$$

$$Z_1 = \frac{sLR_1}{sL + R_1} + R_2 \quad \Rightarrow \quad Z_1 = \frac{sL\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + 1}{\frac{sL}{R_1R_2} + \frac{1}{R_2}} \quad \Leftrightarrow \quad Z_{out} = \frac{sC_{GS}R_S + 1}{sC_{GS} + g_m}$$

$$\Rightarrow \quad R_1 = \left(R_S - \frac{1}{g_m}\right) \qquad R_2 = \frac{1}{g_m} \qquad L = \frac{C_{GS}}{g_m}\left(R_S - \frac{1}{g_m}\right)$$

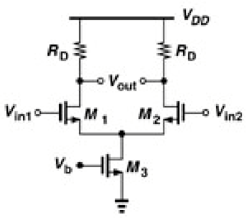
Differential Pair





Differential inputs:
same as common source stage

Differential Pair

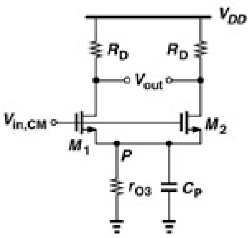


(a)

Common-Mode inputs:

Low frequency A_{CM} with M1-M2 mismatch:

$$A_{CM} = \frac{\Delta g_m R_D}{(g_{m1} + g_{m2}) r_{O3} + 1}$$



(c)

High frequency A_{CM} with M1-M2 mismatch:

$$A_{CM} = \frac{\Delta g_m \left(R_D // \frac{1}{sC_L} \right)}{(g_{m1} + g_{m2}) \left(r_{O3} // \frac{1}{sC_P} \right) + 1}$$

where $C_P = C_{DG3} + C_{DB3} + C_{SB1} + C_{SB2}$