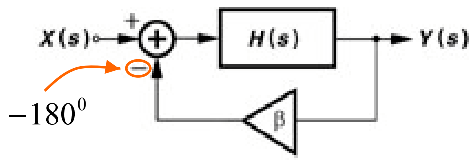
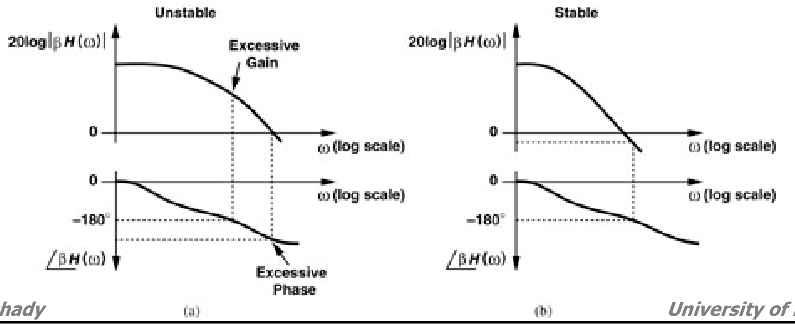
Stability and Frequency Compensation

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1+\beta\,H(s)}$$



Condition for oscillations

$$\beta\,H(j\omega) = -1 \quad \begin{cases} |\beta\,H(j\omega)| = 1 \\ \angle\beta\,H(j\omega) = -180^0 \end{cases}$$



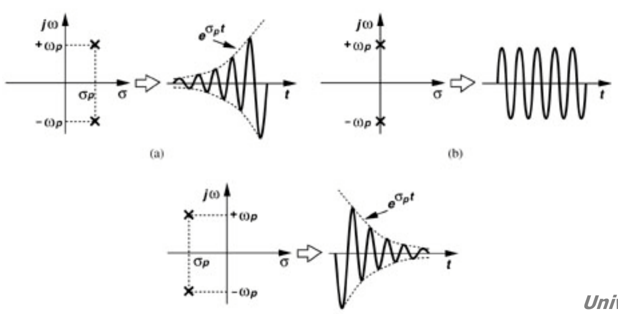
Bode Plot & Root Locus

Bode Plot:

(1) The slope of the magnitude plot changes by
+ 20 dB/dec at every zero frequency
- 20 dB/dec at every pole frequency

(2) For a pole (zero) frequency of ω_m , the phase begins to fall (rise) at $0.1\omega_m$, experiences a change -45^0 ($+45^0$) at ω_m , and a change of -90^0 ($+90^0$) at $10\omega_m$.

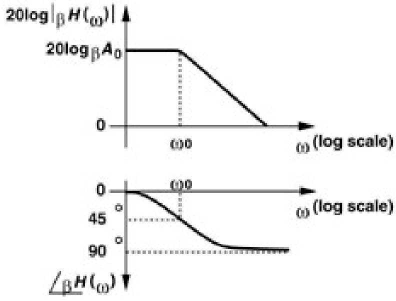
Root Locus: $s_p = \sigma_p + j\omega_p$



One-Pole System

$$H(s) = \frac{A_0}{1 + s/\omega_0}$$

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 + \beta\, H(s)}$$



Bode Plot:

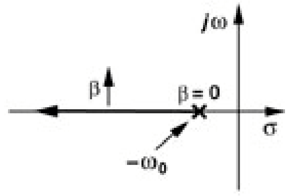
We plot $|\beta\, H(s)|$
and $\angle \beta\, H(s)$ **at** ***s=jω***

A single pole cannot contribute to a phase shift greater than 90°

➡ **the system is unconditionally stable.**

Root Locus:

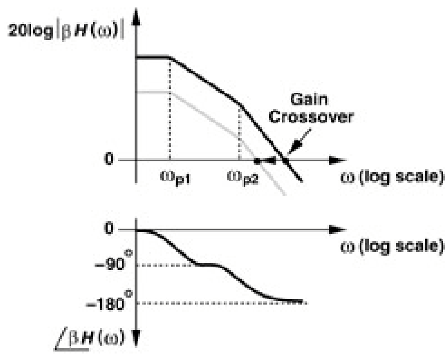
$$s_p = -\omega_0 (1 + \beta\, A_0)$$



We plot the location of the poles as the loop gain varies

Two-Pole System

Bode Plot:



$\beta \downarrow$ ➡ *Gain* $\downarrow$

➡ *No Phase Change*

➡ *More Stable System*

Two-Pole System

Root Locus:

$$H(s) = \frac{A_0}{(1+s/\omega_{p1})(1+s/\omega_{p2})}$$

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1+\beta\,H(s)}$$

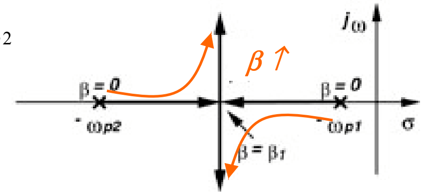
$$\frac{Y(s)}{X(s)} = \frac{A_0}{(1+s/\omega_{p1})(1+s/\omega_{p2})+\beta\,A_0}$$

$$= \frac{A_0\omega_{p1}\omega_{p2}}{s^2+(\omega_{p1}+\omega_{p2})s+(1+\beta\,A_0)\,\omega_{p1}\omega_{p2}}$$

$$s_{1,2} = -\frac{1}{2}(\omega_{p1}+\omega_{p2}) \pm \frac{1}{2}\sqrt{(\omega_{p1}+\omega_{p2})^2-4(1+\beta\,A_0)\omega_{p1}\omega_{p2}}$$

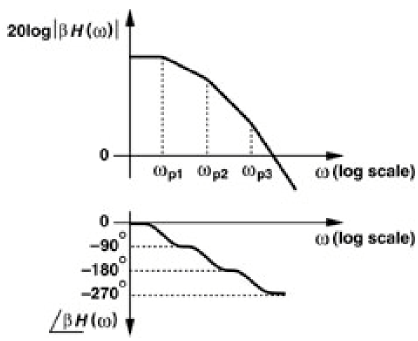
For $\beta = 0$ ➡ $s_{1,2} = -\omega_{p1}, -\omega_{p2}$

$$\beta_1 = -\frac{1}{A_0}\,\frac{(\omega_{p1}-\omega_{p2})^2}{4\,\omega_{p1}\omega_{p2}}$$



Three-Pole System

Additional poles and zeros impact the phase much more than the magnitude

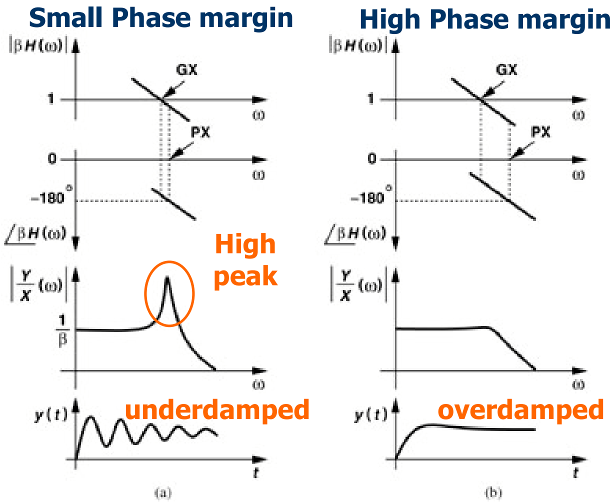


Phase Margin

To ensure stability $|\beta\, H(s)|$ must drop to unity before $\angle\beta\, H(s)$ crosses -180°.

How far should PX be from GX ?

GX: Gain Crossover
PX: Phase Crossover

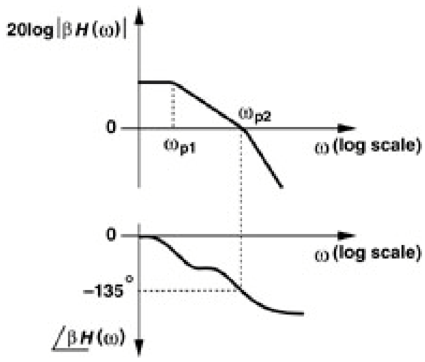


Frequency Response:

Step Response:

Example

A two-pole feedback system is designed such that $|\beta\, H(\omega_{p2})| = 1$ and $|\omega_{p1}| << |\omega_{p2}|$. What is the phase margin ?



Since $\angle\beta\, H(s)$ reaches -135° at $\omega = \omega_{p2}$

➡ The phase margin is equal to 45°.

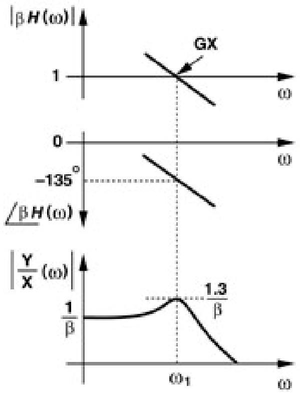
How much phase margin is adequate ?

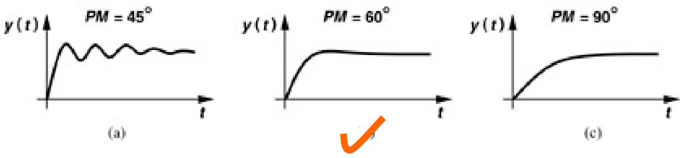
For PM=45° ➡ $\beta H(\omega_1) = 1 \times \exp(-j135)$

$$\left.\frac{Y(s)}{X(s)}\right|_{s=j\omega_1} = \frac{1}{\beta}\frac{1\times\exp(-j135°)}{1+1\times\exp(-j135°)}$$

$$\left.\frac{Y(s)}{X(s)}\right|_{s=j\omega_1} = \frac{1}{\beta}\frac{-\sqrt{2}/2 - j\sqrt{2}/2}{1-\sqrt{2}/2 - j\sqrt{2}/2}$$

$$\left|\left.\frac{Y(s)}{X(s)}\right|_{s=j\omega_1}\right| = \frac{1.3}{\beta}$$

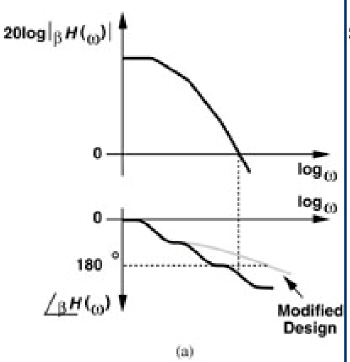




Frequency Compensation

Solution 1

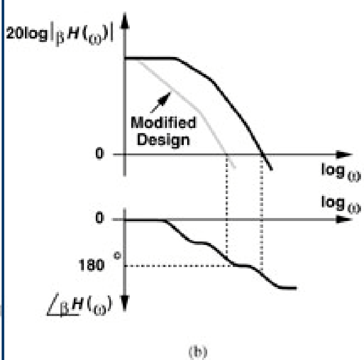
➡ **Modify** $\angle\beta H(s)$



➡ **Poles ↓ ⇒ Stages ↓ ⇒ Gain ↓**

Solution 2

➡ **Modify** $|\beta H(s)|$



➡ **Reduces bandwidth**

Example: Telescopic op amp

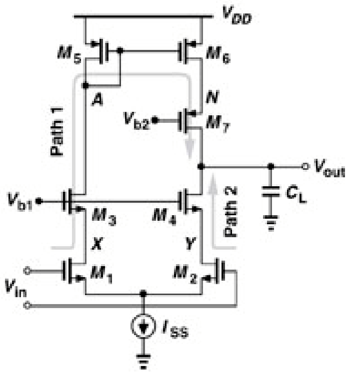
Dominant pole: $\omega_{p,out}$
(highest small signal resistance)

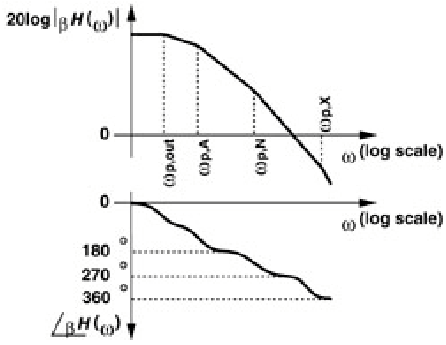
1st Non-Dominant pole: $\omega_{p,A}$
(highest capacitance:)

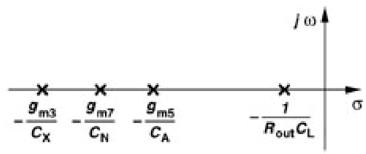
$C_{GS5} + C_{GS6} + C_{DB5} + 2C_{GD6} + C_{DB3} + C_{GD3}$

2nd Non-Dominant pole: $\omega_{p,N}$

3rd Non-Dominant pole: $\omega_{p,X}$

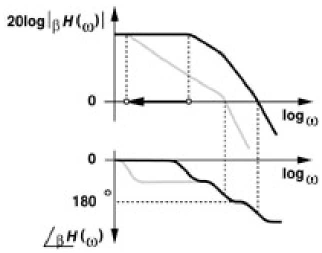




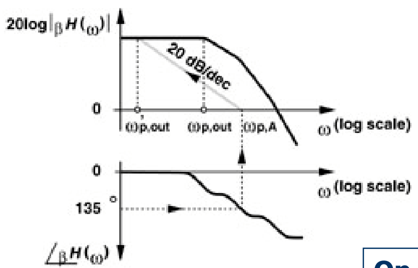


Example: Telescopic op amp

Frequency Compensation:
- lower the frequency of the dominant pole
➡ **increase the load capacitance**



How much $\omega_{p,out}$ must be shifted down ?
Assume:
1- $\omega_{p,A} << \omega_{p,N}$ ➡ $\angle\beta\, H(\omega_{p,A}) = 135°$
2- required PM=45°



➡ **-The new dominant pole:** $\omega'_{p,out}$

-The load capacitance must be increased by a factor $\omega_{p,out} / \omega'_{p,out}$

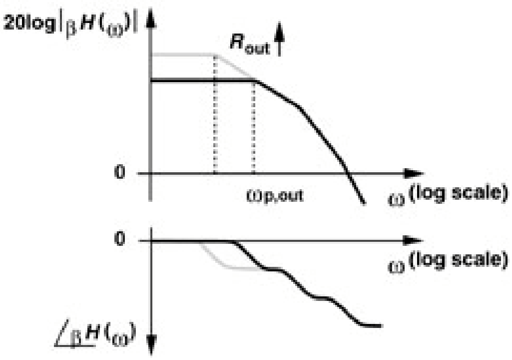
Op-Amp GBW = 1st non dominant pole

Is it possible to compensate using Rout ?

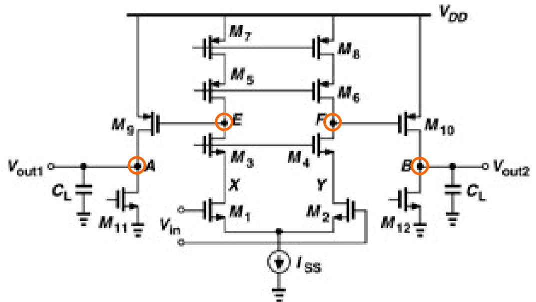
Although,

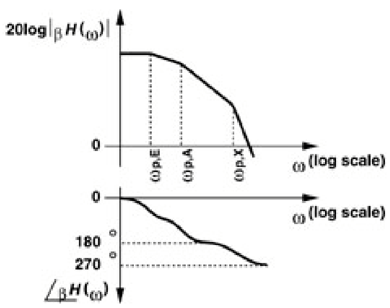
$$\omega_{p,out} = \frac{1}{R_{out} C_L}$$

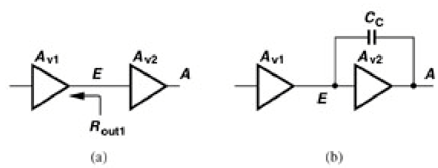
The answer is NO !



Compensation of 2 stage Op-Amps





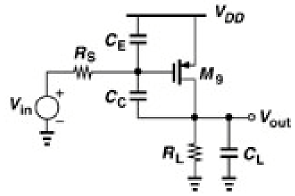


Total capacitance at node E:

$$C_E + (1 + A_{v2}) C_C$$

2nd Stage

Common Source Amplifier:

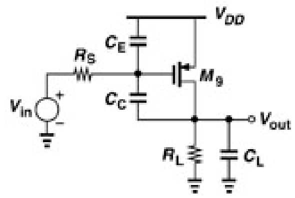


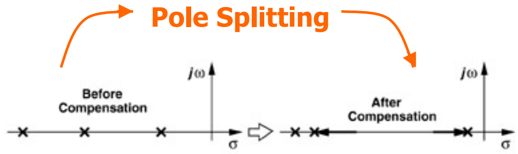
$$\omega_{p1} \approx \frac{1}{R_S\left[(1+g_{m9}R_L)(C_C+C_{GD9})+C_E\right]+R_L(C_C+C_{GD9}+C_L)}$$

$$\omega_{p2} = \frac{R_S\left[(1+g_{m9}R_L)(C_C+C_{GD9})+C_E\right]+R_S C_{GS}+R_L(C_C+C_{GD9}+C_L)}{R_S R_L\left[(C_C+C_{GD9})C_E+(C_C+C_{GD9})C_L+C_E C_L\right]}$$

Pole Splitting

Common Source Amplifier:





Before compensation:

$$\omega_{p1} \approx \frac{1}{R_S\left(C_E+(1+g_{m9}R_L)C_{GD9}\right)} \qquad \omega_{p2} \approx \frac{1}{R_L C_L}$$

After compensation:

$$\omega_{p1} \approx \frac{1}{R_S\left[C_E+(1+g_{m9}R_L)(C_C+C_{GD9})\right]} \qquad \omega_{p2} \approx \frac{g_{m9}}{C_E+C_L}$$